A CAViaR Time-Varying Proportion Portfolio Insurance

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Introduction

Leland and Rubinstein (1976) have shown that an optional asymmetric performance structure can be attained using portfolio insurance strategies. Thanks to dynamic allocation strategies, insured portfolios are protected against large falls by a contractually guaranteed predetermined floor and take partial advantage of market performances.

The Constant Proportion Portfolio Insurance (CPPI) introduced by Black and Jones (1987) and Perold and Sharpe (1988) is a dynamic asset allocation strategy between a risky asset and a risk free one that aims to maintain a constant risk exposure. The covered portfolio benefits from market rallies and is theoretically protected against market crashes by a predefined guaranteed floor. The investor limits her downside risk and participates in a given proportion to the market increase.

The asymmetric portfolio performance structure is defined by one main parameter: the targeted multiple. It defines the risky exposure related to the risky asset exposure. The multiple can be interpreted as the inverse of the maximum drawdown of the risky component on a trading day basis that allows the investor to keep the initial capital safe (*Cf.* Black and Perold, 1992; Poncet and Portait, 1997; Prigent, 2001; Bertrand and Prigent, 2002 and 2005). The higher the multiple, the higher the participation with the risky asset is. In return, the higher the multiple stands, the higher the risk of violating the floor *ceteris paribus*.

Several estimation methods were proposed in the literature to determine the CPPI target multiple. Among these, we can distinguish, on the one hand, unconditional estimation methods based on the historical lowest drawdown (Black and Jones, 1987), the Extreme Value Theory – EVT – (Bertrand and Prigent, 2002), and on the other hand, methods using AutoRegressive specifications of the market volatility (Chen and Chang, 2005).

Despite their numerous drawbacks, these methods are still often used. Actually, they are built using a direct estimation of the unconditional distribution of the risky asset return. Very strong assumptions are needed, such as normality or identical distribution, or independence (Kouretas and Zarangas, 2005).

Following approaches developed within the framework of Value at Risk (VaR), by Chernozhukov and Umanstev (2001) and Engle and Manganelli (2004), we propose in this article a new and more general conditional model for the multiple, using quantile regression.

From a theoretical point of view, the portfolio insurance demand can be both explained within the maximization of the expected utility framework (Basak, 1995 and 2002) or within the so-called Prospect Theory of Kahneman and Tversky (1979) with a loss-averse agent (Berkelaar *et al.*, 2004; Gomes, 2005). The CPPI is optimal when the investor has a decreasing risk aversion (Kingston, 1989). The link between the risk aversion and portfolio insurance is thus clearly established. Being aware that agents have a time-varying risk aversion (*Cf.* Jackson *et al.*, 1972; Campbell, 1999; Coudert and Gex, 2008; Li, 2007) the traditional portfolio insurance could be advantageously adapted to a time-varying framework. Here our strategy keeps a constant risk exposure, and allows us, for instance, to act in a counter cyclical way. The determination of the conditional multiple as a function of the centile is also justified by its interpretation in terms of VaR (considered henceforth as a reference risk indicator) and by works on portfolio allocation under loss constraint and the notion of "disaster" loss that are linked (Roy, 1952; Leibowitz and Kogelman, 1991; Lucas and Klaassen, 1998). In the portfolio insurance framework, we therefore propose to reduce the *ex ante* VaR of a portfolio, in acting on the multiple: reducing it when the VaR tends to increase too much, and inversely, by increasing the leverage when markets are quiet. The set up of the conditional multiple is inspired by usual risk management techniques and by classical VaR control.

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Anchoring our article in these different works, we define our risky asset allocation as a function of the conditional multiple, determined by the level of "disaster" loss estimated thanks to the VaR. The multiple can therefore make the hedging vary according to this risk indicator. Thus, managers aim to have a constant risk exposure as defined by the VaR.

To illustrate the relevance of this new approach, we evaluate the performances of the portfolio determined by several specifications of the conditional multiple and of various portfolios managed with an unconditional target multiple. Our analysis focuses on a portfolio managed and invested for the risky part in the S&P500. We study the performance of four particular autoregressive specifications for the conditional centile[1] and three advanced versions[2]. The sample is composed of daily returns of the S&P500 index[3] observed from January 4th 1993 to January 16th 2008. We also provide an extensive backtest of the selected method on the *Dow Jones* Index since January 1937.

If classical portfolio insurance methods can soften a certain bear shape of the market, our approach seems, however, in the test period, less costly in terms of return and risk.

This article is organised as follows. Section I presents the general framework of cushioned management. Section II analyses the set up of the variable target multiple determined by a quantile regression method, and also presents four autoregressive specifications of the centile. In section III, we compare, in the American stock market, the performances obtained with our conditional method with those of the main unconditional determination method of the target multiples. Section IV concludes.

I. Cushioned Portfolio Evidences

Portfolio insurance allows investors to recover, at maturity, a given proportion of their initial capital. One of the standard portfolio insurance methods is the Constant Proportion Portfolio Insurance (CPPI). This strategy allocates dynamically a risky asset and a non-risky one to guarantee a predetermined floor.

The management of cushioned portfolio follows a dynamic strategic allocation. The floor, denoted F_t, is the *minimum* value of the portfolio which is acceptable for an investor at maturity. The value of the covered portfolio, denoted V_t, is invested in a risky asset and a non-risky asset. The proportion invested in the risky asset varies relatively to the amount invested in the non-risky asset, in order to insure at any time the guaranteed floor. Hence, even if the market is downward sloping at the investment horizon T, the portfolio will keep at maturity a value at least equal to the floor, (*i.e.* a predetermined percentage of the capital deposit at the beginning). The final guaranteed theoretical value cannot be superior, obviously, to the invested value initially capitalised at the non-risky rate, denoted by $[V_0 Exp(rT)]$. The cushion, denoted c_t, is defined as the spread, which can vary across time, between the portfolio value and the value of the guaranteed floor, such as:

$$c_t = V_t - F_t. \quad (1)$$

The cushion is thus the theoretical *maximum* amount which can be lost over a period without reducing the guaranteed capital.

The ratio, at each time, between the risky asset and the cushion corresponds to the targeted multiple, denoted m_t. The amount invested in the risky asset is determined in multiplying the cushion by the multiple. The multiple reflects the *maximal* exposure of the portfolio. The cushioned management strategy aims to keep a constant proportion of risk exposure. The position of the risky asset, denoted e_t, has to be proportional to the cushion. Thus, we have at any time:

$$e_t = m_t \times c_t. \quad (2)$$

The fluctuating multiple moves around its target value, this is the reason why a third parameter is introduced, the tolerance, denoted τ, to determine whether the portfolio should be rebalanced. If, after a variation of the risky asset, the remaining multiple, denoted m_t^* moves away from its target value by a superior percentage defined by the tolerance parameter, there will be adjustments (thus, transaction costs), then $\forall t \in [0, \ldots, T]$, such as:

$$m_t^* \in [m_t \times (1-\tau), m_t \times (1+\tau)]. \quad (3)$$

The main issue regarding the management of the cushion is the determination of the multiple, m_t. Hence, if the risky asset drops, the value of the cushion remains, by definition, superior or equal to zero, since the portfolio stays at a value in theory superior or equal to the floor. However, in case of a drop of the risky underlying asset, the higher the multiple, the more the cushion and the exposure tend rapidly to zero. Nevertheless, transactions are not executed continuously. Then if the quotation of the risky asset drops suddenly, before the manager can rebalance his portfolio, the cushion aims to absorb a shock inferior or equal to the inverse of the upper limit of the multiple.

II. Setting of the Conditional Multiple by the Quantile Regression Method

The target multiple can be interpreted as the inverse of the *maximum* drawdown that can be borne, over a unit period, by the covered portfolio before its risky part can be rebalanced.

Our goal is to set a new conditional estimation method of the target multiple in a cushioned portfolio framework[4].

To perform this aim, we use a hedging quantile approach similar to a conditional Value-at-Risk coverage[5]. According to this probabilistic coverage approach, the multiple can be estimated by the inverse of the first conditional centile of the risky asset return distribution, augmented by the $\alpha - th$ quantile of the excess return with respect to this centile, denoted $Q(\alpha)$. The target multiple can then be formally written as:

$$m_t = \left| C_t(r_t;\beta_t) + Q_t(\alpha) \right|^{-1} \quad (4)$$

with:

$$C_t(r_t;\beta_t) = \beta_{0,t} + \beta_{1,t} \times C_{t-1}(r_{t-i};\beta_t) + \sum_{i=2}^{p} \beta_{i,t} \times l(x_t)$$

where $C_t(r_t;\beta_t)$ is the first centile of the daily risky asset conditional return distribution, r_t is the periodical risky part return of the covered portfolio, β_t is the vector of dimension (p + 1) of the unknown parameters of the conditional centile function, x_t are the components of the (p – 1) explanatory variable vector, l(.) is the first order lag operator, and $Q_t(\alpha)$ stands for the excess return quantile when the conditional centile is overtaken. The presence of the autoregressive term in the expression of $C_t(r_t;\beta_t)$ gives some flexibility to the centile value across time.

The potential *maximum* drawdown of the risky asset return is therefore estimated, at each period of time, and we add the excess return observed when the quantile is overtaken, denoted by $Q_t(\alpha)$, which is computed over the estimation period such as:

$$\widehat{Q}_t(\alpha) = \underset{t=[1,\ldots,\Gamma]}{Min}\left[r_t - C_t(r_{t-1};\beta_t)\right] \quad (5)$$

where $C_t(r_{t-1};\beta_t)$ is the risky asset return centile estimated with the selected conditional model and Γ is the total number of observations corresponding to an estimation period.

Following Engle and Manganelli (2004), we use for the general model the last known return as a lagged variable[6] for the conditional multiple determination.

To model the conditional target multiple, we also consider four particular autoregressive specifications. The first specification corresponds to an adaptive model which can be written such as:

$$C_{At}(r_t;\beta_{1,t}) = C_{At-1}(r_{t-1};\beta_{1,t}) + \beta_{1,t}\left[1 + Exp\left\{.5 \times \left[r_{t-1} - C_{At-1}(r_{t-1};\beta_{1,t})\right]\right\}\right]^{-1} - .01 \quad (6)$$

where r_{t-1} is the risky return part of the covered portfolio on the last day.

The intuition of this specification is the following: as long as the daily return is not inferior to the first centile estimation, the conditional multiple can increase by a small amount, which allows us to benefit from potential rises of the risky part of the insured portfolio. On the contrary, when the first centile is exceeded, the variable multiple has to decrease, thus the portfolio is protected against larger and larger falls of the risky asset. This model adapts itself to its past errors and allows the reduction of the probability that the target multiple is consecutively underestimated, but it does not guarantee that the target multiple is not overestimated.

The second specification is a symmetric absolute model, it is defined such as:

$$C_{SVAt}(r_t;\beta_t) = \beta_{1,t} + \beta_{2,t} \times C_{SVAt-1}(r_{t-1};\beta_t) + \beta_{3,t} \times |r_{t-1}| \quad (7)$$

with $\beta_t = (\beta_{1,t}, \beta_{2,t}, \beta_{3,t})$.

In this model, the conditional multiple symmetrically reacts to positive or negative returns of the underlying risky asset. The asymmetric slope model is the third specification used to estimate the functional form of the conditional multiple. It can be written as such:

$$C_{PAt}(r_t;\beta_t) = \beta_{1,t} + \beta_{2,t} \times C_{PAt-1}(r_{t-1};\beta_t) + \beta_{3,t} \times r^+_{t-1} + \beta_{4,t} \times r^-_{t-1} \quad (8)$$

$$\text{where: } \begin{cases} r^+_{t-1} = \max(0, r_{t-1}) \\ r^-_{t-1} = -\min(0, r_{t-1}) \end{cases}$$

with $\beta_t = (\beta_{1,t}, \beta_{2,t}, \beta_{3,t}, \beta_{4,t})$.

Thus, the conditional multiple varies differently according to positive or negative return levels.

The fourth and last autoregressive specification used in this article is an "*Indirect* GARCH(1,1)" model (see Engle and Manganelli, 2004). Its algebraic expression is given by:

$$C_{IGt}(r_t;\beta_t) = \left\{\left[\beta_{1,t} + \beta_{2,t} \times \left[C_{IGt-1}(r_{t-1};\beta_t)\right]^2 + \beta_{3,t} \times r^2_{t-1}\right]\right\}^{1/2} \quad (9)$$

with $\beta_t = (\beta_{1,t}, \beta_{2,t}, \beta_{3,t})$.

This model is correctly specified if underlying data are generated by a GARCH(1,1) model[7] with an Independently and Identically Distributed residual.

To improve the conditional multiple estimation, we propose three extensions for each of the predefined autoregressive specifications, based on the successive addition of the following lagged exogenous variables: the ratio between the highest and the lowest within day prices (*range*), the implied volatility of the risky asset return and the volume of the traded stocks within the last few days.

We have chosen to link the conditional multiple to advanced market indicators available for most of the quoted values. Thus, the *range* is computed thanks to the daily *maxima*; it is traditionally considered as a better volatility estimator than the simple classical and commonly used empirical volatility (*Cf.* Parkinson, 1980) because it is based on the important "true" price process (highest and lowest prices of a quotation day), rather than on arbitrary references of calendar time (closing prices). The implied volatility, extracted from the options market, is also a classical advanced turbulence market indicator and an instantaneous volatility indicator (Corrado and Miller, 2005); it also has some predictive power for the future realized volatility. At last, the within-day traded stocks volume is a useful indication of the market information flow (*Cf.* Clark, 1973 for an underlying explanation of the mixture distributions and the subordinated volatility process, and, Ané and Geman, 2000, for several alternatives of proxies of the information on the market such as volume, number, intensity of transactions...).

Conditional multiples estimated using these additional advanced indicators are called herein « Advanced Conditional Multiples».

For each selected autoregressive specification, parameters of the variable multiple model are estimated using the quantile regression method[8]. Parameters of the first conditional centile are thus obtained as the solution of the following optimisation program:

$$\beta_t^* = \underset{\beta_t \in IR^n}{ArgMin} \quad \{RQ(\beta_t)\} \qquad (10)$$

with:

$$RQ(\beta_t) = \sum_{t=1}^{T} \left\{ \left[.01 - I_{\{r_t < C_t(r_t;\beta_t)\}} \right] \times \left[r_t - C_t(r_t;\beta_t) \right] \right\}$$

where $RQ(.)$ is the value associated to the quantile regression objective function; $I_{\{.\}}$ stands for the identity function and $C_t(.)$ is a particular autoregressive specification of the first conditional risky asset return – *Cf.* equations (6), (7), (8) and (9).

In the next section, we analyze the estimates of previous conditional autoregressive specifications in the American stock market. Then, we compare relative performances of insured portfolios managed with conditional and unconditional multiples.

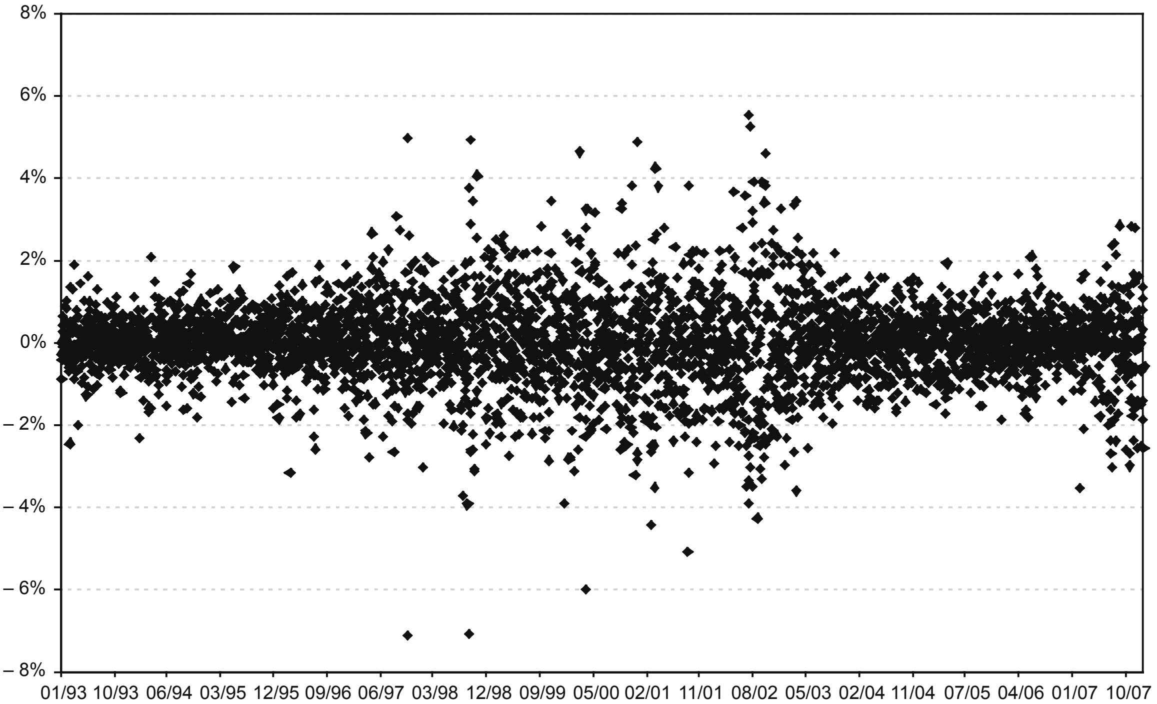
III. Empirical Results

To illustrate our approach, we have chosen to compose a portfolio invested for its risky part in the American index S&P500. The non-risky part is invested at the American daily money market rate. Our sample[9] begins on the 4th of January 1993 and ends on the 16th January 2008. After having estimated the several conditional centile specifications presented in section III, we have chosen the model that minimizes, in the estimation period, the value associated with the quantile regression objective function[10]. The addition of several leading indicators to the base model allows us to improve the estimation quality of the conditional multiple. The 2,787 first prices, volumes, *ranges* and *VIX* are used to estimate the various multiples[11] and the last points to test the advanced conditional multiple model[12].

To associate to the floor violation a probability inferior or equal to 1% over a four and a half year data sample (*i.e.* 1,135 observations), we should add to the first selected conditional centile estimation, the estimated value of the quantile associated to a 0.09% probability level of the difference between the risky asset return and the first conditional centile, we have chosen to estimate it thanks to the largest observed spread over the estimation period between the S&P500 return and the first conditional centile.

We first present estimation results obtained for the four basic models of the first conditional centile: that is to say, the symmetric absolute model (model 1), the asymmetric

Figure 1. *S&P500* Daily Returns Overestimation and Test Periods



Source: *Bloomberg*, daily data, *S&P500* closing prices from 01/05/1993 to 01/16/2008; computations by the authors.

slope model (model 2), the "Indirect GARCH" model (model 3) and the adaptive model (model 4)[13].

Considering table 1, which presents our estimation results, we notice that the autoregressive term parameter is significant whatever the model considered[14], this one is associated to a high value for the three former models. A high volatility dependence is thus illustrated, even in the distribution tails. We also notice that the first centile exceedance probability level is, by construction, nearly 1% over the estimation period, this probability is negligible over the test period.

Figures A.1, A.2 and A.3 in the appendix present, over the estimation period, the S&P500 opposite daily return and the opposite of the first centile estimated respectively with the symmetric absolute model, the asymmetric slope model and the "*Indirect GARCH*" model,. For these three models, centile evolutions, represented over figures A.1, A.2 and A.3, seem visually well estimated: centile exceedances do not draw any cluster and they have various amplitudes. These three models thus seem to be well adapted to describe the S&P500 centile dynamic. The conditional symmetric absolute value and "Indirect GARCH" centile models are dependent on absolute risky asset return variations. However, analysing, in one hand, results from the symmetric absolute value model and the "Indirect GARCH" model, and, in the other hand, results from the asymmetric slope model, the first S&P500 conditional centile seems to be characterized by asymmetric behaviour. The impact of a large negative return contributes an important part to the evolution of the first conditional centile. Positive returns seem to have a less significant influence on the quantile dynamic. When the S&P500 return is strongly negative, the centile decreases. But positive returns have no major influence on it. The asymmetric slope model also seems to be well adapted to a high volatility period when important positive and

Table 1. Estimation Results for the four *S&P500* Basic Centile Models from 09/2003 to 01/2008

The four presented conditional centile models are respectively given as:

$$\begin{cases} C_{SVAt}(r_t;\beta_t) = \beta_{1,t} + \beta_{2,t} \times C_{SAVt-1}(r_{t-1};\beta_t) + \beta_{3,t} \times |r_{t-1}| \\ C_{PAt}(r_t;\beta_t) = \beta_{1,t} + \beta_{2,t} \times C_{PAt-1}(r_{t-1};\beta_t) + \beta_{3,t} \times \max(0,r_{t-1}) + \beta_{4,t} \times [-\min(0,r_{t-1})] \\ C_{IGt}(r_t;\beta_t) = \{\beta_{1,t} + \beta_{2,t} \times [C_{IGt-1}(r_{t-1};\beta_t)]^2 + \beta_{3,t} \times r^2_{t-1}\}^{1/2} \\ C_{At}(r_t;\beta_{1,t}) = C_{At-1}(r_{t-1};\beta_{1,t}) + \beta_{1,t}\left[1 + Exp\{.5 \times [r_{t-1} - C_{At-1}(r_{t-1};\beta_{1,t})]\}\right]^{-1} - .01 \end{cases}$$

where r_{t-1}, is the one day lagged daily return the risky underlying asset.

	Model 1 - Absolute Value -	Model 2 - Asymmetric Slope -	Model 3 - *Indirect GARCH* -	Model 4 - Adaptive -
β_1	**.07**	**.13**	**.10**	**.24**
Standard -deviation	.03	.04	.04	.11
P-Statistic	.83%	.00%	1.00%	1.30 %
β_2	**.96**	**.91**	**.94**	–
Standard -deviation	.02	.03	.01	
P-Statistic	.00%	.00%	.00%	
β_3	**.09**	(– .07)	(.27)	–
Standard -deviation	.04	.06	.42	
P-Statistic	1.21%	11.76%	25.95%	
β_4	–	**– .16**	–	–
Standard -deviation		.05		
P-Statistic		.00%		
RQ	**98.34**	**94.46**	**97.36**	**99.45**
Hits In-sample	.93%	.96%	.97%	1.11%
Hits Out-of-sample	.00%	.00%	.00%	.00%
Cumulative Loss (USD)	25.83	25.15	25.64	27.40
Maximum Loss (USD)	4.58	4.08	4.58	4.65

Source: *Bloomberg*, daily data, *S&P500* closing price from 01/05/1993 to 01/16/2008; computations by the authors. The 2,787 first prices are used to estimate the various multiples and the last points to test the conditional multiple model. Parameters between brackets are non-significant at a threshold of 5%. Significant values presented in this table have been computed only taking into consideration significant terms. *RQ*(.) is the value associated to the quantile regression objective function.

negative returns are consecutive but when the conditional quantile remains low.

The adaptive centile model appears, over the sample period used in this article, to have the lowest explanatory power of the dynamics of the first conditional centile of the S&P500 conditional centile (the quantile regression objective function is here associated to the highest value). This last model has the advantage to reduce the repeated and consecutive overestimation of the first conditional centile but, it cannot avoid, by construction, the underestimation of the extreme quantile. Thus we observe in figure A.4 in the appendix (which represents the opposite of the first centile estimated by the adaptive model and the opposite of the S&P500 daily return) that the centile estimated by the adaptive method increases regularly and slowly according to time except when the underlying asset return is below the centile. This new information is then taken into consideration and the centile decreases suddenly. The adaptive specification does not allow the investor for the anticipation of large variations, but just follows them. This particular behaviour leads us to not retain this specification.

Over the sample, the symmetric absolute value model is associated to a lower objective function value[15]. However, this model seems to be particularly well adapted to the high volatility period. We have therefore chosen to retain this last specification as the reference model to establish the conditional multiple.

To improve the estimation of the conditional multiple, we add, in a second step, several leading indicators to the asymmetric slope specification of the conditional centile. Advanced asymmetric slope specifications, denoted C_{PA1t}, C_{PA2t} and C_{PA3t}, thus depend – besides significant variables – respectively on the ratio between the highest and the lowest daily price observed during the last S&P500 quotation day, the implied volatility extracted from the American stock options market, and on the daily S&P500 stocks exchanged volume. Figures A5, A6, A7 in appendix 1 represent the time-evolutions over the estimation period of the first conditional centile obtained for each of the three advanced asymmetric slope specifications.

We have also represented in table 2 (see below) estimations results obtained, over the sample period, of the best advanced specification for the conditional centile: that is to say, the asymmetric autoregressive specification that uses the negative return part and the *range* (specification denoted C_{PA1t}).

Let us recall that, for the asymmetric conditional specification, the variable target multiple is empirically determined such as, with t = [1, ..., T]:

$$\widehat{m}_t(\beta_t) = -\left[C_{PA1t}(r_t;\beta_t) + \widehat{Q}_t(\alpha)\right]^{-1} \qquad (11)$$

with:

$$\widehat{Q}_t(\alpha) = \underset{t=[1,\ldots,\Gamma]}{Min}\left[r_t - C_{PA1t}(r_{t-1};\beta_t)\right]$$

where $C_{PA1t}(.)$ is the value associated to the conditional centile model specification depending on the negative return part and on the underlying risky asset return *range*, $\widehat{Q}_t(\alpha)$ is the maximal exceedance from the first conditional centile observed over the estimation period, with a significance level α and Γ as the overall observations during the estimation period.

The advanced asymmetric conditional multiple, computed according to the S&P500 *range* fluctuates, over the test period, between 8.1 and 10.5 with an average of 9.7 and a standard deviation of 0.4. The average conditional multiple is therefore coherent with traditional unconditional multiples used by managers (between 4 and 9); it can also reach, in some market configurations, a level comparable to the unconditional multiple estimated by Extreme Value Theory. The *maximum* unconditional multiple that we consider in our study is arbitrarily set at 13. This value is much more important than multiples traditionally used by practitioners, but it is a coherent value according to the worst possible loss estimated by extreme value in the American market (see Bertrand and Prigent, 2002). This value is, however, high and unrealistic, for usual practical applications; this "extreme" value allows us to set a limit in the following. Nevertheless, the insured portfolio performance *via* a conditional multiple depends obviously on the evolution of the risky underlying asset price.

Cappiello *et al.* (2005) methodology can be used to visualize returns comovements of two portfolios. This representation called the "Comovement Box" is illustrated in our case by figure 3. The Y-axis represents the conditional probabilities associated to the quantile of the returns of the portfolio managed with a conditional multiple, conditional to the fact of the realizations of the other compared portfolio returns. The X-axis represents the probability α associated to quantiles of the portfolio returns to compare with. Figure 3 thus illustrates the probability that the return of the portfolio managed with a conditional multiple is inferior to its own α-quantile conditional to the fact that the return of the portfolio with a multiple of 13 is also below its own α-quantile, when the α is inferior to 50%. When α is superior to 50%, we then represent the probability that the return of the conditional portfolio is superior to its quantile when the return of the unconditional portfolio is above its quantile.

Three particular cases are represented for illustrative purposes. The perfect correlation between the returns of the two portfolios is represented by a horizontal line that takes the value 1; the probability that the first portfolio return belongs to the same quantile as the second portfolio return is in this case always 100%. Thus, when the return of the unconditional portfolio is extreme, the conditional portfolio return is also extreme. The "triangle" – in grey – represents a perfect independence between the returns of the two portfolios under analysis; actually, the probability that the two portfolio returns belong to the same quantile is here exactly equal to α%. Extreme comovements are, in this case, not linked: the unconditional density cannot be distinguished from the conditional density, and the portfolio conditional probability is independent of the unconditional portfolio realizations. Finally, the horizontal line that takes a null value represents a perfect negative correlation between the two portfolios returns; the probability that

Table 2. Estimation Results for the Advanced Asymmetric *S&P500* Centile Model from 09/2003 to 01/2008

We recall the three advanced conditional centile models given as:

$$C_{PAit}(r_t;\beta_t) = \beta_{1,t} + \beta_{2,t} \times C_{PAit-1}(r_{t-1};\beta_t) + \beta_{3,t} \times [-\min(0, r_{t-1})] + \beta_{4,t} \times z_{t-1}$$

For the models C_{PA1t}, C_{PA2t} and C_{PA3t}:

- z_{t-1} is the ratio between maximum and minimum prices during the last *S&P500* quotation day.
- z_{t-1} is the implied volatility extracted from the American options market on stocks.
- z_{t-1} is the traded volume of the *S&P500* constituent stocks.

	C_{PA1t} - Range -	CA_{PA2t} - VIX -	CA_{PA3t} - Volume -
β_1	**2.31**	**– 7.31**	**– 0.45**
Standard -deviation	.18	3.53	.19
P-Statistic	.00%	1.92%	.87%
β_2	**.85**	(– .27)	**.95**
Standard -deviation	.03	.28	.06
P-Statistic	.00%	17.17%	.00%
β_3	**– .18**	(– .20)	(.04)
Standard -deviation	.05	.17	.05
P-Statistic	.03%	10.98%	20.02%
β_4	**– .40**	**3.60**	**.03**
Standard -deviation	.03	.83	.02
P-Statistic	.00%	.00%	6.82%
RQ	**90.07**	**90.51**	**95.97**
Hits In-sample	.93%	.93%	.97%
Hits Out-of-sample	.20%	.60%	.00%
Cumulative Loss (USD)	21.57	18.58	26.08
Maximum Loss (USD)	4.17	4.07	4.68

Source: *Bloomberg*, daily data, closing prices, *minimum* and *maximum* daily prices, *S&P500* daily traded volume and implied volatility extracted from the American options market on American stocks (VIX) from 01/05/1993 to 01/16/2008; computations by the authors. The 2,787 first prices, volume, range and VIX are used to estimate the various multiples and the last points to test the advanced conditional multiple model. Parameters between brackets are non-significant at a threshold of 5%. Significant values presented in this table have been computed only taking into consideration significant terms. *RQ*(.) is the value associated to the quantile regression objective function.

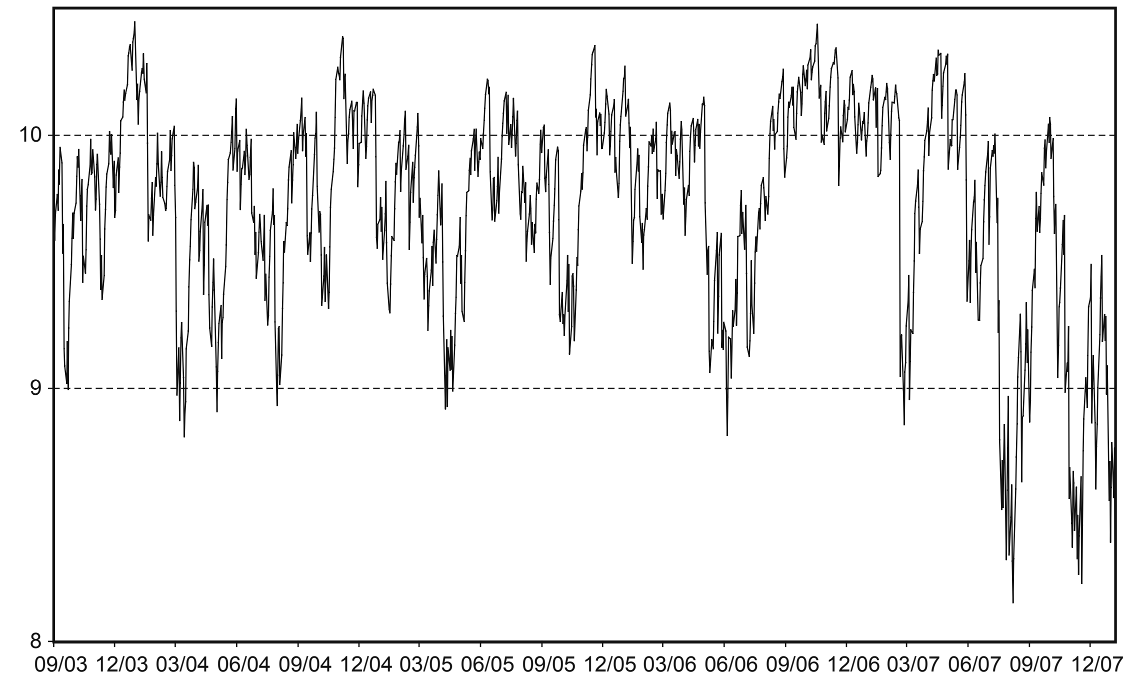
the return of the first portfolio is below its own quantile when the second portfolio is also null; thus portfolios react in an anti-symmetric way and their movements vary in opposite ways.

In figure 3, we visualize comovements between two portfolio returns: one managed with a conditional multiple and the other with an extreme unconditional multiple of 13; we notice that when the returns are very low (the first decile of the returns distribution) or very high (the five upper centiles) the conditional portfolio returns are weakly linked to the portfolio with an unconditional multiple of 13, but correlations between these two portfolios are important for median values of the defined returns. The behaviour of one of the conditional portfolios is different from that of the unconditional portfolio in particular market situations: most of the time, the two portfolios have the same behaviour, but are very different in extreme events (when protection is used).

The protected portfolio driven by a conditional multiple and computed according to the *range* and with the asymmetric specification is strongly dependent on the S&P500 evolution, except when displaying "abnormal" negative returns.

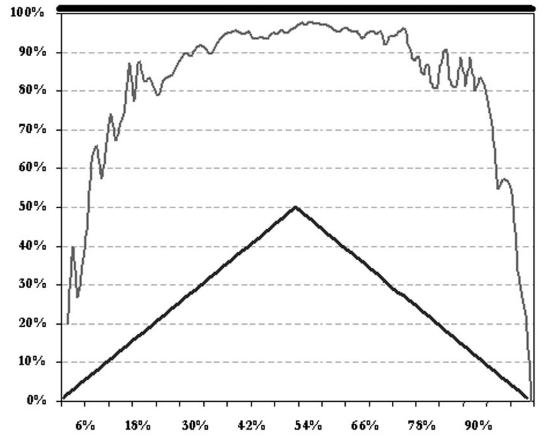
Based on the previous estimate of the conditional multiple, we can now evaluate the interest of our approach, comparing performances of the strategies using a traditional CPPI unconditional multiples approach and the conditional multiple of the S&P500 from September 2003. For the sake of clarity, we choose to compare the portfolio managed with a conditional multiple based on the advanced asymmetric specification (equation 11) with three traditional unconditional strategies: the unconditional strategy that reached the best value at the end of the test period (*ex post maximal* return), the strategy managed with an unconditional multiple determined by the Extreme Value Theory, and the *ex ante* less risky

Figure 2. Evolution of the Advanced Asymmetric Conditional Multiple estimated on the S&P500 from 09/2003 to 01/2008



Source: *Bloomberg*, daily data, closing prices, *minimum* and *maximum S&P500* prices from 01/05/1993 to 01/16/2008; computations by the authors. The 2,787 first prices and range are used to estimate the various multiples and the last points to test the advanced conditional multiple model.

Figure 3. Comovement Box between the returns of the portfolio managed with the conditional multiple and the returns of the CPPI with a constant multiple of 13 on the *S&P500* from 09/2003 to 01/2008



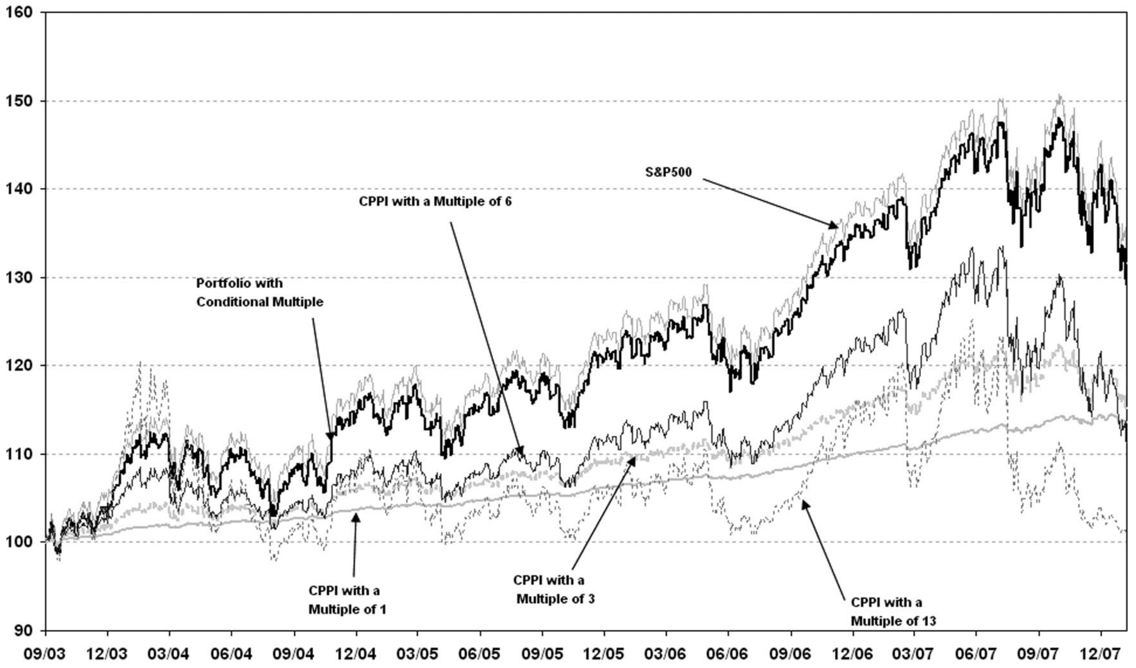
Source: *Bloomberg*, daily data, closing prices, *minimum and maximum S&P500* prices from 01/05/1993 to 01/16/2008; computations by the authors following Cappiello *et al.* (2005). The 2,787 first prices and range are used to estimate the various multiples and the last points to test the advanced conditional multiple model. In this figure, are represented (see the grey line), the conditional quantile of the returns of the cushioned portfolio managed with a conditional multiple knowing that the quantile of the returns of the CPPI with a multiple of 13. The "triangle" notices a perfect independence between the returns of the two cushioned portfolios. The horizontal line associated to the unit value points out a perfect correlation between the two portfolio returns. Finally, the horizontal line associated to the null value shows a null correlation between the two portfolio returns.

unconditional cushioned strategy. These strategies are in our sample, the portfolios with the unconditional multiple equal respectively to 3, 13 and 1.

Figure 4 represents the evolution of these cushioned portfolios on the S&P500 from September 2003. The cushioned strategy using a conditional multiple dominates unconditional cushioned strategies most of the time. At the end of the test period, the portfolio managed with the conditional advanced asymmetric multiple is associated to the highest value; it brings an average of 6.05% by year over the test period, when the best unconditional strategy gives only 3.26% *per* year, and these unconditional cushioned strategies reach on average 1.90% *per* year over the same period.

The cushioned portfolio risky underlying asset, the S&P500 index, yields over the same period 6.49% by year

Figure 4. Values of the Insured Portfolios with Conditional and Unconditional Multiples (1, 3, 6 and 13) on the *S&P500* from 09/2003 to 01/2008



Source: *Bloomberg*, daily data, closing prices, *minimum* and *maximum S&P500* prices from 01/05/1993 to 01/16/2008; computations by the authors. The 2,787 first prices and range are used to estimate the various multiples and the last points to test the advanced conditional multiple model. In this figure, are represented: the *S&P500* values (in grey), values associated to the four cushioned portfolios with constant multiples of 1, 3, 6 and 13 (a thick continuous line, fine line, bold dotted line and fine dotted line), and the value associated to the portfolio managed with a conditional multiple (in bold continuous line).

over the test period. The absolute cost of insurance by the advanced conditional multiple method is thus, over this period, lower than the costs associated to all unconditional methods for cushioned portfolios (*i.e.* the difference between the underlying asset return and the return of insured portfolios set to 0.44% for the insured portfolio, managed according to the conditional multiple against respectively 3.23% and 6.27% for insured portfolios with constant unconditional multiples of 3 and 13).

The three compared portfolio return volatilities are inferior to the underlying asset volatility. The CPPI portfolio managed with a multiple of 1 is, of course, associated to the lowest volatility over the test period. Actually, this one is the best to attenuate the fluctuations of the risky underlying asset. By contrast, the portfolio, managed according to an unconditional multiple of 13 is associated over the sample period to a relatively high annualized volatility over the whole sample period (15.61%).

The portfolio managed according to the advanced asymmetric conditional multiple (equation 11) also dominates over the test period the portfolios managed with unconditional multiples according to the risk-return ratio, with a Sharpe ratio of .28 against respectively .12 and – .16 for the traditional CPPI portfolios with a target multiple fixed to 3 and 13. Besides, we notice that all the skewness associated to the returns of the insured portfolios are here significantly different from zero, and the normality tests confirm that their return distributions are not normal.

Although the Sharpe ratio remains a reference indicator (Sharpe, 1994), its only use, for assessing the relevancy of the strategy, is questionable, in particular, because of the skewness created by insured strategies. Although the second order moment can always be interpreted as a dispersion measure associated to the underlying distribution, the non-normality of insured portfolio returns leads us to a cautious interpretation of the results and to complete the comparative analysis based on the Sharpe ratio using complementary performance measures. We present Sortino ratios (using as a reference threshold the mean of the returns), Omega measures (using an arbitrary reference threshold of zero) and third-order Kappa measures (to take into account the skewness of the studied strategies), these measures generalise the Sharpe ratio in the case of non-normal distributions.

Indeed, the Sortino ratio is a modified Sharpe ratio, the volatility being replaced by the semi-volatility at the denominator (Sortino and van der Meer, 1991). In case of asymmetry in the return distribution, the Sortino ratio reduces the excess return by a risk measure more linked to the investor potential loss (return inferior to its mean value). The Omega performance measure, introduced by Keating and Shadwick (2002), is defined as the ratio between the surface under the returns cumulative distribution function below and the one above a given return threshold. A higher Omega measure means that the return density is more "important" above the threshold than below. In fact, this ratio takes into consideration the skewness of returns, but also the *kurtosis* of returns of the strategy under study, through a synthetic measure using main moment characteristics of the return distribution. Finally, the performance measure Kappa, introduced by Kaplan and Knowles (2004), uses a more general risk measure; the Sortino ratio is equivalent to the second order Kappa measure, and the Omega measure is equal to the unity plus the first order Kappa measure. The Kappa measure associated to order *n* is the ratio between the excess return at a given threshold, and the Lower Partial Moment (LPM) of order *n* with respect to the same threshold.

The comparison between the Sortino ratio (Sortino and van der Meer, 1991), the Omega measure (Keating and Shadwick, 2002) and the Kappa measure (Kaplan and Knowles, 2004) allows us to rank the portfolio managed according to the conditional multiple in the first quartile among every tested cushioned strategy (the strategy managed with a conditional multiple and the thirteen unconditional strategies managed with a constant multiple – from 1 to 13). The comparison of the return density functions associated to insured portfolios managed with conditional and unconditional multiples (see figures 5 and 6) underlines the fact that the return distributions of the unconditional portfolio managed with an unconditional multiple of 13 has a fatter left tail than the left tail associated with the returns of the conditional portfolio (managed with a conditional multiple). The CPPI associated with a target multiple of 3 is characterised by a very low probability of loss (inferior to 1.8%) and a very poor participation rate in case of the rise of the risky underlying asset (the probability that the return overcomes 1.5% is nearly null). These observations are illustrated by figure 7.a. Finally, the portfolio managed according to the conditional advanced asymmetric multiple model (equation 11) leads to a very fast adaptation of the risky asset exposure to market fluctuations (as is illustrated in figures 8.a and 8.b.). The $VaR_{99\%}$ conditional strategy is, over the test period, inferior to the $VaR_{99\%}$ computed for the CPPI, with a constant multiple has been determined by the Extreme Value Theory (– 2.35% against – 3.08%). It happens to be the case that the insured portfolio with a conditional multiple decreases when the risky underlying asset has negative returns -in particular after consecutive positive returns- but its exposure is adapted very quickly when the market goes up and finally comes back among

Table 3a. Cushioned Portfolio Strategy Characteristics on the *S&P500* from 09/2003 to 01/2008

S&P500 (09/2003-01/2008)	Risky Asset	Variable Multiple	Multiple 1	Multiple 3	Multiple 6	Multiple 13
Return	6.49%	6.05%	2.95%	3.26%	2.47%	.22%
Volatility	12.24%	12.14%	1.25%	4.81%	11.20%	15.61%
VaR 99%	– 2.35%	– 2.35%	– .25%	– 1.03%	-2.29%	– 3.08%
Skewness	-.33	– .36	– .46	– .69	– .97	– 1.16
P-Statistic	.00%	.00%	.00%	.00%	.00%	.00%
Kurtosis	4.83	4.89	6.54	7.89	8.99	9.99
P-Statistic	.00%	.00%	.00%	.00%	.00%	.00%
Jarque-Bera	180.03	193.08	632.44	1221.78	1877.78	2563.73
P-Statistic	.00%	.00%	.00%	.00%	.00%	.00%
Kolmogorov-Smirnov	.49	.49	.50	.49	.49	.49
P-Statistic	.00%	.00%	.00%	.00%	.00%	.00%
Anderson-Darling	7.96	8.02	12.86	18.21	19.18	16.06
P-Statistic	.00%	.00%	.00%	.00%	.00%	.00%
Sharpe	.31	.28	.23	.12	– .02	– .16
Sortino	.05	.05	.21	.06	.02	.01
Omega	1.10	1.10	1.50	1.13	1.05	1.02
Kappa	.03	.03	.14	.04	.01	.00

Source: *Bloomberg*, daily data, closing prices, *minimum* and *maximum S&P500* prices from 01/05/1993 to 01/16/2008; computations by the authors. The 2,787 first prices and ranges are used to estimate the various multiples and the last points to test the advanced conditional multiple model. Returns and Volatilities are annualized. The VaR in each column corresponds to the daily return empirical quantile associated to a 99% confidence level. The skewness and *kurtosis* P-statistics are related to Pearson parametric tests.

the best ranks (against other unconditional strategies – see on figure 3 the period beginning for example from December 2003 to March 2004).

The suggested approach for a conditional portfolio insurance with a conditional multiple seems, therefore – over this test period and with this particular risky asset (S&P500) – more efficient (*i.e.* less costly, for a limited risk) than for traditional unconditional CPPI methods.

After having detailed the results on historical series, which are directly linked to the "true" evolution of market configurations (trends, reversals...), we have completed the analysis performing the very same comparisons using extensive stationary bootstrap simulations (Politis and Romano, 1994) from a corresponding series on S&P500 returns. A stationary bootstrap allows us, in fact, to keep partially the dependence structure between the returns series under study[16]. We thus compare portfolios managed with the conditional multiple and traditional unconditional CPPI with a fixed multiple in table 3.b. The portfolio managed with a conditional multiple is the first according to the Sharpe ratio over 500 simulations and belongs to the first distribution quartile for every other risk measure. The portfolio managed with a conditional multiple is ahead of most of the traditional CPPI with an unconditional multiple[17].

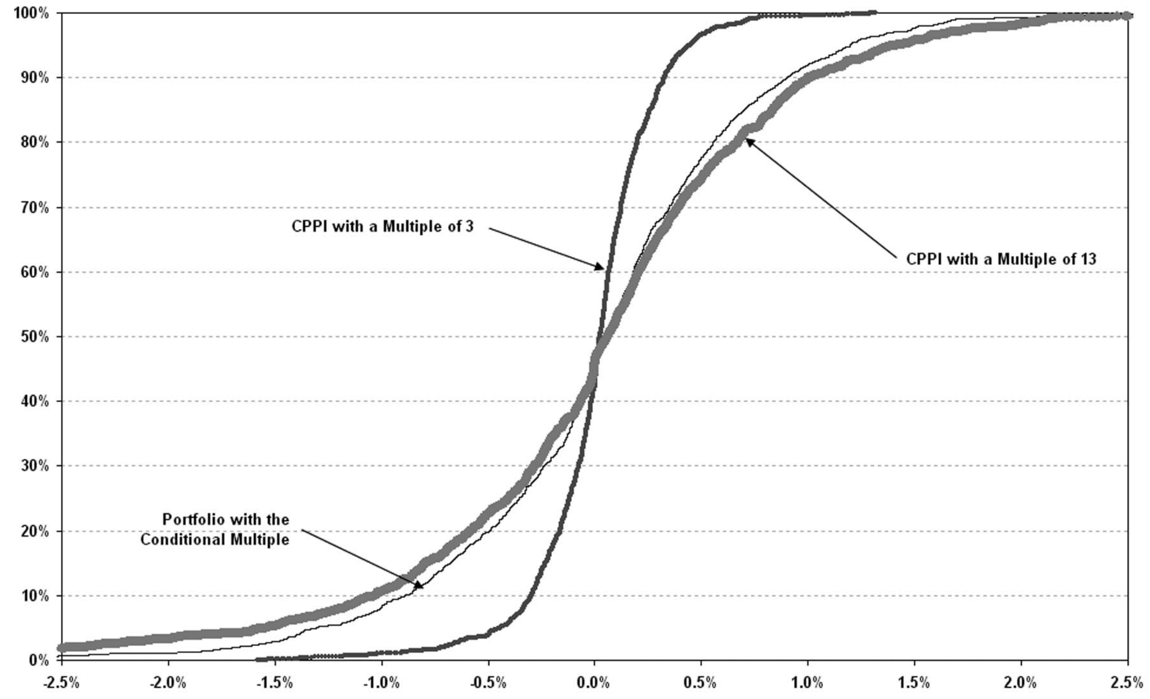
As a final illustration, we have also dynamically estimated the conditional multiple, in the American equity market from the first January 1937, for an insured portfolio whose risky part is invested in the Dow Jones Index. The series considered here (the DJIA from 1937 and 2008) is thus particularly long (18,554 daily returns) and it extends the analysis of the conditional strategy performances in various market configurations (bullish, bubble burst, sudden and violent crisis, uncertainty period, high volatility and state of low volatility...). Moreover, this series takes into account the evolution of most of the main stocks of the main equity market around the world; it therefore has such a "universal" and representative behaviour. Then and above all, to reinforce the robustness of our conclusions, several strategies have been initiated from the series start point and aggregated results are presented in table 4. To build this table, we have actually, from the first sample date, investigated what would have been the performances of a set of portfolios launched every 20 days for a period of three years. Thus, in Table 4, there are a large number of tested strategies, with several

Table 3b. Cushioned Portfolio Strategy Characteristics on *S&P500* Bootstrapped Simulated Series from 2003 to 2008

S&P500 (09/2003-01/2008)	Risky Asset	Variable Multiple	Multiple 1	Multiple 3	Multiple 6	Multiple 13
Return	6.49%	4.62%	2.99%	3.59%	3.41%	2.58%
Volatility	12.24%	10.28%	1.50%	5.65%	13.43%	28.42%
VaR 99%	– 2.35%	– 2.01%	– .24%	– 1.01%	– 2.39%	– 4.48%
Skewness	– .33	– .25	– .17	– .27	– 1.19	– 1.53
P-Statistic	.00%	.00%	.00%	.00%	.00%	.00%
Kurtosis	4.83	14.34	8.15	37.60	152.89	257.63
P-Statistic	.00%	.00%	.00%	.00%	.00%	.00%
Jarque-Bera	$1.80\ 10^2$	$3.05\ 10^5$	$6.31\ 10^4$	$2.83\ 10^6$	$5.32\ 10^7$	$1.53\ 10^8$
P-Statistic	.00%	.00%	.00%	.00%	.00%	.00%
Kolmogorov-Smirnov	.49	.49	.50	.49	.48	.47
P-Statistic	.00%	.00%	.00%	.00%	.00%	.00%
Anderson-Darling	7.96	8.02	12.86	18.23	19.20	16.07
P-Statistic	.00%	.00%	.00%	.00%	.00%	.00%
Sharpe	.31	.45	.22	.16	.05	.00
Sortino	.05	.04	.19	.06	.03	.02
Omega	1.10	1.12	1.42	1.14	1.09	1.12
Kappa	.03	.03	.12	.03	.01	.01

Source: *Bloomberg*, daily data, closing prices, *minimum* and *maximum S&P500* prices from 01/05/1993 to 01/16/2008; computations by the authors. The 2,787 first prices and ranges are used to estimate the various multiple and the last points to test the advanced conditional multiple model. The strategy characteristics are calculated using 500 simulations of 3,921 daily returns based on a stationary bootstrap (see Politis and Romano, 1994): artificial series are composed with *S&P500* random blocks of daily returns determined using a geometric probability law defined by a parameter of .9. Statistics presented here are the averages of the statistics computed for each strategy over all simulation. Returns and Volatilities are annualized. The VaR in each column corresponds to the daily return empirical quantile associated to a 99% confidence level. The skewness and kurtosis P-statistics are related to Pearson parametric tests.

Figure 5. Cumulative Distribution Function of Cushioned Portfolios with Conditional and Unconditional Multiples on *S&P500* from 09/2003 to 01/2008



Source: *Bloomberg*, daily data, closing prices, *minimum* and *maximum S&P500* prices from 01/05/1993 to 01/16/2008; computations by the authors. The 2,787 first prices and ranges are used to estimate the various multiples and the last points to test the advanced conditional multiple model. The plotted lines point out the empirical Cumulative Distribution Functions (cdf) of the returns of strategies under comparison. The thin continuous line represents the cdf associated with returns of the portfolio managed with a conditional multiple. The bold continuous line represents the cdf associated to the returns of the CPPI managed according to an unconditional multiple of 13; the third line corresponds to the cdf of the returns associated to the CPPI with a fixed multiple of 3.

starting points (overall 890 consecutives starts, for 3,710 estimations using 18,554 observations). Model parameters are estimated every week within a three-year initial capital guarantee. The conditional advanced asymmetric multiple (equation 11) varies over this period around an average value of 8.08.

In table 4, we see that the portfolio managed thanks to the conditional advanced asymmetric multiple model was monetarized only one time over the whole period, after 50 years and 8 months. We also notice that only CPPI with fixed multiples inferior or equal to 4 are never totally monetarized over the studied period; all other portfolios are monetarized at least once (during the crisis of October 1987). The portfolio with the conditional multiple has on average, over this long sample, the highest Sharpe ratio (which is coherent with the results previously exposed on the S&P500 in a more recent period). The mean excess annualized return (and also the maximum drawdown) of the conditional strategy is superior to that corresponding to the CPPI with a fixed multiple of 7, but its return volatility is close to that of the CPPI with a fixed multiple of 5. We see over this long sample that the conditional strategy dominates, according to the performance, the volatility and the Sharpe ratio, a CPPI strategy with a fixed multiple of 6.

Thus, comparing the conditional strategy results to all unconditional strategies, we have found, *ex post*, some strategies with a fixed multiple which are better (for some *criteria*) than the strategy managed with a variable conditional multiple. But this is an *ex post* choice: which multiple must be chosen *ex ante*? We have tried to answer this question, proposing a systematic flexible (variable) approach, which increases the exposure when the risk associated to the risky underlying asset is low and that limits the risky asset exposure when the risky asset is in a turbulent situation. We indeed adapt in the simplest way the traditional risk management approach to the portfolio insurance framework. The extended analysis of this systematic portfolio strategy using a conditional multiple (determined *ex ante*) gives actually interesting results because it overtakes or is close to the best unconditional strategy determined *ex post* (when the return risky asset evolution is already known) according to the main financial classical strategy comparison *criteria*.

IV. Conclusion

The target multiple is one of the main essential parameters of cushioned portfolio insurance. Rather than

Table 4. Coverage Analysis of Portfolios managed with Conditional and Unconditional Multiples Corresponding to several monthly starts on the *Dow Jones Index* from 01/1937 to 01/2008

DJIA (01/1937-01/2008)	Monetarization Definitive Date	Number of potential loss	Underestimation of the potential loss	Mean Annualized	Average
Variable Multiple	10/19/1987	1	11.27%	1.43%	5.70%
Multiple 1	Never	0	–	.23%	.97%
Multiple 2	Never	0	–	.35%	1.99%
Multiple 3	Never	0	–	.48%	3.06%
Multiple 4	Never	0	–	.62%	4.18%
Multiple 5	10/19/1987	1	2.61%	.84%	5.40%
Multiple 6	10/19/1987	1	5.94%	1.04%	6.67%
Multiple 7	10/19/1987	1	8.32%	1.35%	8.06%
Multiple 8	10/19/1987	1	10.11%	1.63%	9.46%
Multiple 9	10/19/1987	1	11.50%	1.92%	10.95%
Multiple 10	10/19/1987	1	12.61%	2.21%	12.37%
Multiple 11	10/19/1987	1	13.52%	2.41%	13.84%
Multiple 12	10/19/1987	1	14.28%	2.43%	15.07%
Multiple 13	10/19/1987	3	14.92%	2.01%	16.06%

Source: *Bloomberg*, weekly data, closing prices, *minimum* and *maximum Dow Jones* prices from 10/02/1928 to 01/16/2008; computations by the authors. Every week, model parameters are dynamically estimated. Excess Return are computed against the risk free rate and annualized. A three-year horizon is used. Statistics presented above are the average characteristics of all the computed portfolios (total of 890), with a three year guarantee horizon, which are launched every months from January 1937 for each strategy (every line of the table). The date for which the portfolio is monetarized is equivalent to the monetarization of every portfolio launched during the last three years (from October 1984 to October 1987).

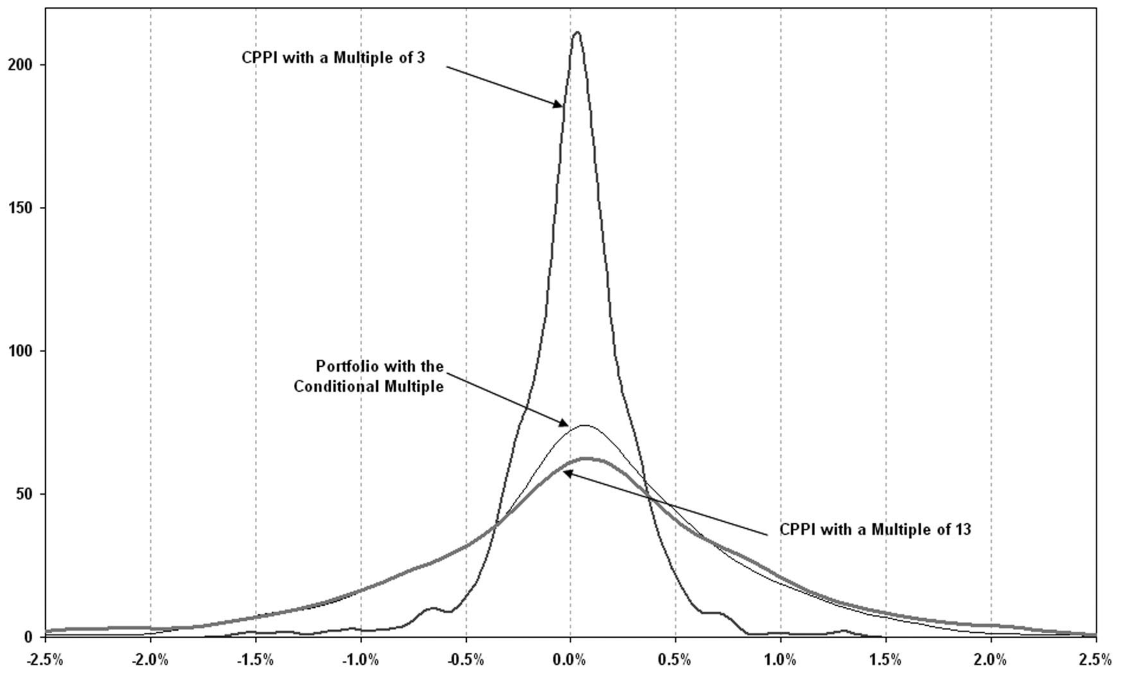
trying to determine an *optimal* multiple from the historical maximum drawdown or from a particular specification of the volatility, we have chosen to develop in this article a new specification method for the multiple, based on an autoregressive extreme conditional quantile model using the quantile regression framework initiated by Koenker and Basset (1978). We have also developed an advanced version for the conditional multiple thanks to the use of lagged leading indicators available for the majority of quoted assets.

The portfolio strategy insured by the advanced asymmetric conditional multiple actually dominates most of the time, out of sample, portfolios insured by an unconditional fixed multiple, in particular those estimated according to the Extreme Value Theory (see Bertrand and Prigent, 2002).

Among future research development, it seems interesting to extend the analysis framework to more general portfolio insurance strategies. This could be achieved using the mixed risk aversion utility maximisation function approach under optimal guaranteed constraint with polynomial options (see Cox and Huang, 1989; Caballé and Pomansky 1996; Macovschi and Quittard-Pinon, 2006; and Prigent and Tahar, 2006). It could also be interesting to generalize our approach combining the quantile regression technique with other dynamic estimation methods such as conditional parametric quantile regression, regressions on extremes, statistic expansions and quantile regression with exponential smoothing (see Chernozhukov, 2005 and Taylor, 2008-a and 2008-b). A first approach was developed by Gouriéroux and Jasiak (2008) who establish sufficient quantiles combination conditions through the Dynamic Additive Quantile models.

A relative performance analysis for the different conditional multiple estimation methods would be useful in this framework. The predictive power of several conditional multiple estimation methods can be further tested. But to achieve this goal, an appropriate and proper comparison framework for portfolio insurance strategies has to be developed, taking into consideration the agents' preferences, the dependence within the insured strategy starting point (strategy path dependence), the estimation frequency, the underlying risky asset and the coverage horizon. ■

Figure 6. Estimation of the Probability Distribution Function of Cushioned Portfolios with Conditional and Unconditional Multiples on *S&P500* from 09/2003 to 01/2008



Source: *Bloomberg*, daily data, closing prices, *minimum* and *maximum S&P500* prices from 01/05/1993 to 01/16/2008; computations by the authors. The 2,787 first prices and ranges are used to estimate the various multiples and the last points to test the advanced conditional multiple model. Non-parametric Probability Distribution Functions (pdf) are estimated using a Gaussian kernel using a cross-validation *criterion* (see Silverman, 1986). The thin continuous line represents the pdf associated to returns of the portfolio managed with a conditional multiple. The bold continuous line represents the pdf associated to the returns of the CPPI managed according to an unconditional multiple of 13; the third line corresponds to the pdf of the returns associated to the CPPI with a fixed multiple of 3.

Thanks

We thank Thierry Chauveau, Thierry Michel, Paul Merlin, Jean-Philippe Médecin and Jean-Luc Prigent for their kind help and advice. The third author thanks the Europlace Institute of Finance for financial support. We acknowledge the participants of the *Journées de Micro-économie Appliquée* (Nantes, 2006), the *Journée d'Econométrie Financière Avancée* (Paris, 2006) and the *Annual International Conference of AFFI* (Bordeaux, 2007) for comments and suggestions. We also acknowledge two anonymous referees for their helpful remarks, as well as the editor. The usual disclaimer applies.

1 The conditional centile is estimated thanks to an adaptive model, an absolute value model, an asymmetric slope model, an "Indirect GARCH(1,1)" model.

2 The three advanced versions are based on exogenous variables.

3 Data that we need were not available before this moment.

4 The manager (for various practical purposes: to decrease transaction costs, to satisfy *maximum* exposition constraint, operational constraints…) often chooses in a discretionary way a target multiple inferior or equal to the upper limit of the multiple defined by a model, stress tests or by a third party guarantee (see Poncet and Portait, 1997). In this article and for evaluation issues, we have chosen to follow a systematic approach: the conditional multiple is thus determined to keep a constant risk exposure defined by the VaR.

5 This approach was initially proposed for a CPPI with a constant multiple (see Bertrand and Prigent, 2005).

6 An abnormally negative return can indicate the beginning of a turbulent period in the market. The conditional multiple is expected to decrease when the last known return is negative and to increase when this one is positive. The variation effect of the risky asset return has an asymmetric impact on the target multiple.

7 *Cf.* Engle and Bollerslev (1986).

8 Every estimation was computed using MatLab 7.8.

9 All the data that we need were not available before this moment.

10 *Cf.* equation (10).

11 The Volatility Index® (denoted VIX®) is the implied volatility of the *S&P500* index options over the next 30 days. The VIX is provided by the Chicago Board Options Exchange. (CBOE) It is a weighted blend of prices for a range of options on the *S&P500* index. It takes as inputs the current market prices for all out-of-the-money calls and puts for the front month and second month expirations. Introduced in 1993, it is often considered as a good proxy of short term market anticipation for the instantaneous future volatility of the *S&P500* Index (*Cf.* Corrado and Miller, 2005).

12 The estimation period, presents important negative returns of the risky asset such as, for example: the 1997-1998 Asiatic crisis, the burst of the internet bubble in 2000, or the 11th September 2001.

13 These models are respectively denoted: $C_{SVAt}(\beta_t)$, $C_{PAt}(\beta_t)$, $C_{IGt}(\beta_t)$ and $C_{At}(\beta_t)$.

14 The parameter in front of the auto regressive term is denoted $\beta_{1,t}$ for the adaptive model and $\beta_{2,t}$ for every other model.

15 Estimation results of the models presented are, however, too similar to be *ex ante* distinguished. We do not pretend to be able to make a definitive categorical differentiation between these different models but we propose some other alternative which gives reasonable, and good and relatively similar results. In the following, for presentation purposes, we only retain one specification, on *ad hoc criteria* and an arbitrary adjustment.

16 The aim of the drawings and random recombinations of block of various sizes (the block size is here defined by a geometric probability law) is to create a large number of artificial series, preserving at best the auto-correlation structure of the original real series.

17 We have also performed these estimations on the CAC40 Index from its creation; in the French market, general conclusions of the analysis are very similar.

Figure 7. QQ-plot of Returns associated to Portfolio managed with Unconditional Multiples (3 and 13) with respect to Returns associated to the Portfolio Managed with the Conditional Multiple on the S&P500 from 09/2003 to 01/2008

Figure 7a. Fixed Multiple of 3 *versus* Conditional

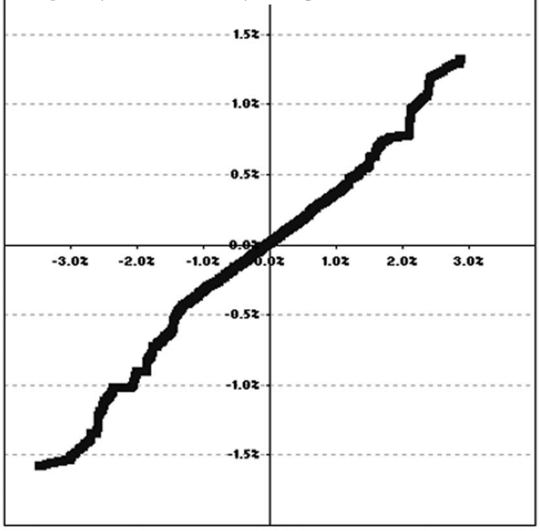
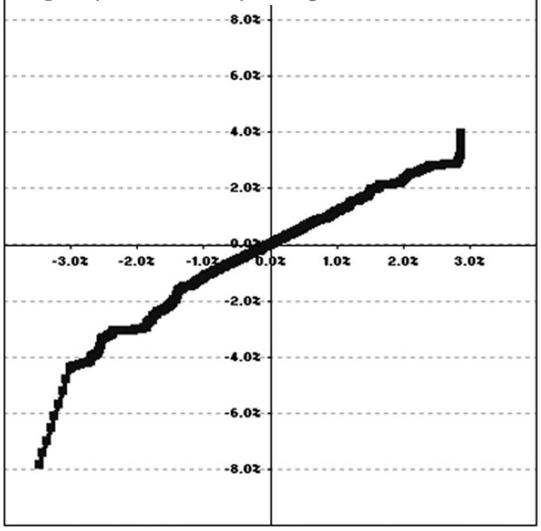


Figure 7b. Fixed Multiple of 13 *versus* Conditional



Source: *Bloomberg*, daily data, closing prices, *minimum* and *maximum S&P500* prices from 01/05/1993 to 01/16/2008; empirical cumulative distribution function (cdf); computations by the authors. The 2,787 first prices and ranges are used to estimate the various multiples and the last points to test the advanced conditional multiple model. On the left side: return distribution of the *CPPI* with a multiple of 3 with respect to the return distribution of the cushioned portfolio with the conditional multiple. On the right side: return distribution of the *CPPI* with a multiple of 13 with respect to the return distribution of the cushioned portfolio with the conditional multiple.

Figure 8. Returns associated to the Portfolio Managed with the Conditional Multiple with respect to Returns associated to Portfolio Managed with Unconditional Multiples (3 and 13) on the *S&P500* from 09/2003 to 01/2008

Figure 8a. Conditional *versus* Multiple of 3

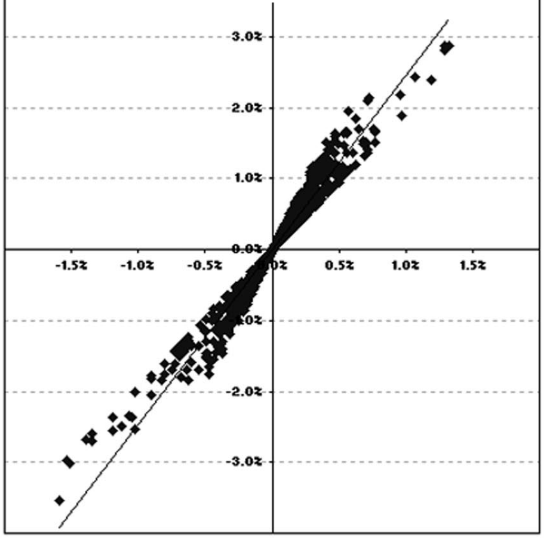
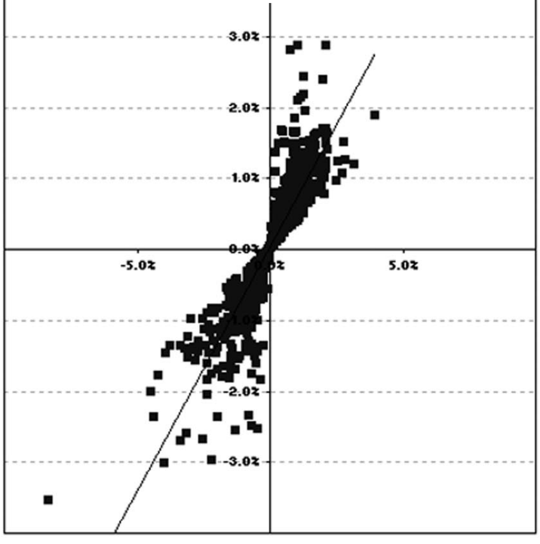


Figure 8b. Conditional *versus* Multiple of 13



Source: *Bloomberg*, daily data, closing prices, *minimum* and *maximum S&P500* prices from 01/05/1993 to 01/16/2008; computations by the authors. The 2,787 first prices and ranges are used to estimate the various multiple and the last points to test the advanced conditional multiple model. On the left side: returns associated with the cushioned portfolio with the conditional multiple with respect to returns associated with the *CPPI* with a multiple of 3. On the right side: returns associated with the cushioned portfolio with the conditional multiple with respect to returns associated with the *CPPI* with a multiple of 13.

APPENDIX 1

Figure A1. Opposite of the Centile estimated by Model 1 based on a Symmetric Absolute Value Specification and the opposite of *S&P500* daily returns

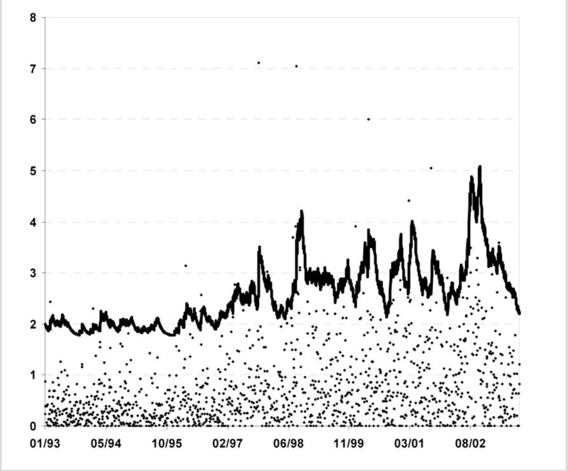


Figure A2. Opposite of the Centile estimated by Model 2 based on an Asymmetric Slope Specification and the opposite of *S&P500* daily returns

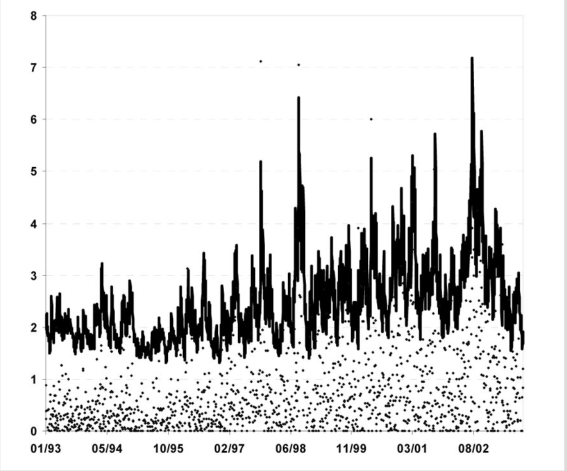


Figure A3. Opposite of the Centile estimated by Model 3 based on an *Indirect GARCH* Specification and the opposite of *S&P500* daily returns

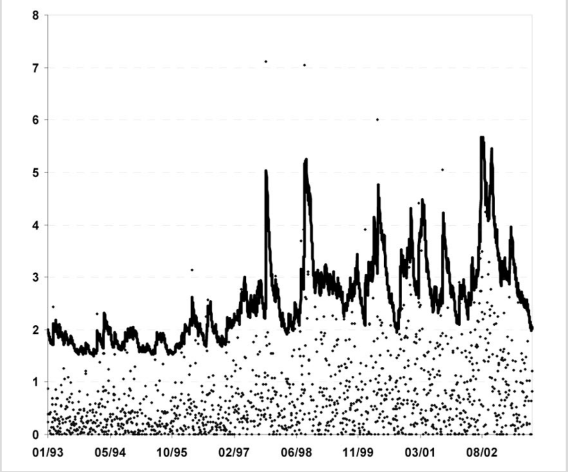
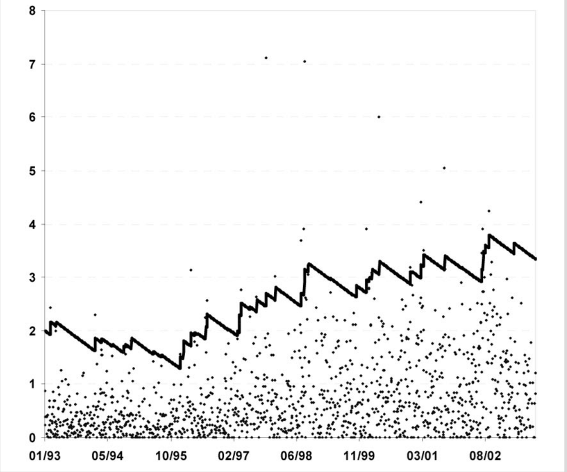


Figure A4. Opposite of the Centile estimated by Model 4 based on an Adaptive Specification and the opposite of *S&P500* daily returns



Source: *Bloomberg*, daily data, *S&P500* closing prices from 01/05/1993 to 08/10/2003; computations by the authors. Over these figures are represented: the opposite of S&P500 returns and centile estimations according to the four basis model - see equations (6), (7), (8) and (9).

Figure A5. Opposite of the Centile estimated by Model 5 based on the Range and the opposite of *S&P500* daily returns

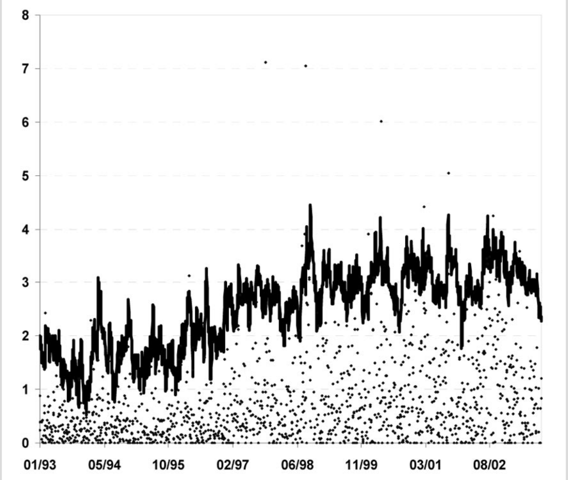


Figure A6. Opposite of the Centile estimated by Model 6 based on the Implied Volatility and the opposite of *S&P500* daily returns

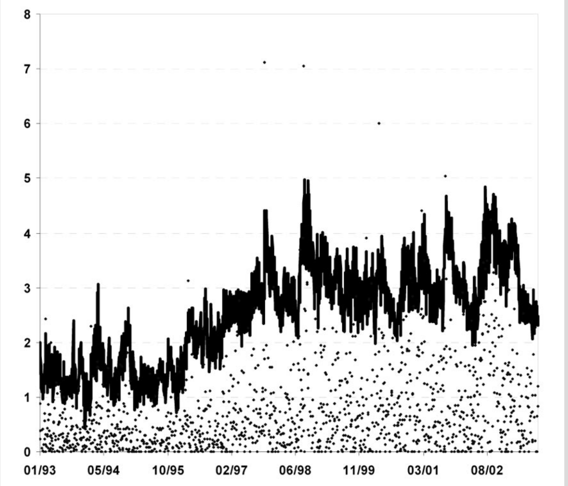
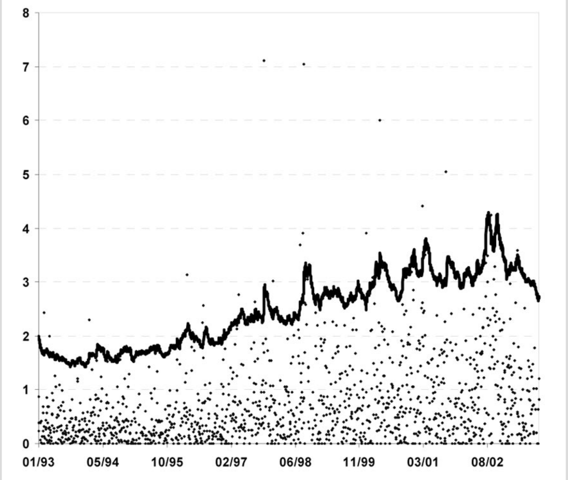


Figure A7. Opposite of the Centile estimated by Model 7 based on the volume and the opposite of *S&P500* daily returns



Source: *Bloomberg*, daily data, closing prices, *minimum* and *maximum* daily prices, *S&P500* daily traded volume and implied volatility (VIX) extracted from the American options market on American stocks from 01/05/1993 to 08/10/2003; computations by the authors.

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