The term structure of crude oil futures prices: A principal component analysis

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I Introduction

This article is centered on the movements of the crude oil futures prices' curve. The term structure of futures prices describes the relationships between the spot price and futures prices for different delivery dates. It is supposed to resume all the information needed to hedge positions on the physical market, to undertake arbitrage operations or to support investment decisions. Thus, understanding the behavior of the prices' curve is a prerequisite for the use of derivatives instruments. Moreover, permanent factors are required for managing price risk or pricing derivatives.

In the American crude oil market, the concept of term structure is all the more important that there are futures contracts for very far delivery dates: up to seven years. In order to identify the crude oil prices' movements, to test their stability and to examine whether or not they are independent of the futures contracts' maturity, we use the method of the principal component analysis, that takes historical data on movements in the prices and attempts to define a set of components or factors that explain these movements.

In the case of commodities, Cortazar and Schwartz (1994) show that three factors explain the dynamic behavior of the term structure of copper prices. These movements correspond to parallel shifts of the curve (level factor), relative shifts (steepness factor) and deformations (curvature factor). Tomalsky and Hindanov (2002) extend the empirical work of these two authors to the petroleum market. They propose a principal component analysis in multicommodity and seasonal markets. Lastly, Borovkova (2003) uses principal component analysis as a way to detect market transitions, from backwardation to contango and back. Our study corroborates these previous works, in the case of the crude oil market. Moreover, we reach two conclusions concerning the evolution of the prices dynamic on a long period of time, and the influence of the maturity on this dynamic. These conclusions constitute the main contributions of this article.

The first conclusion is related to the analysis of the evolution of prices' behavior on a long period of time. It is the first study on such a long period: thirteen years, from 1989 to 2002. Therefore, it enables the investigation of the eventual structural nature of prices' movements. Empirical tests show that everything being equal, the same factors (level, steepness and curvature) can be identified on the whole period and that there are little changes in their respective intensity from one period to another. Even an a priori specific period like the first Gulf War dœs not stand out from the others. Thus, there is a stable structure of prices' dynamic.

The second conclusion concerns the influence of maturity on prices' dynamic. This study includes very far maturities: until seven years for the period 1999-2002. Two principal component analyses are compared on this period: the first relies on futures contracts having a maturity ranging from one to eighteen months, the second takes account of all available maturities, from the first to the 84th month. Empirical tests show that increasing the maturity significantly influences the behavior of the prices' curve. Indeed, the first factor looses some of its explicative power, whereas the second and – more marginally – third factors gain in importance. Consequently, even if there are permanent factors in the futures prices' dynamic, as the crude oil futures market comes to fruition, the introduction of long-term contracts changes slightly this dynamic. This transformation implies that risk management on long-term maturities is more complicated that on short-term ones.

These statements are important for several reasons. Firstly, such a study constitutes a useful prerequisite for the elaboration of term structure models of commodity prices, especially for the crude oil market. Secondly, understanding the prices' dynamic is important for risk management. Indeed, principal component analysis can be used in order to quantify the risks of a portfolio, for Value At Risk analysis for example. Such a tool is all the more interesting that the dynamic factors are stable.

The remainder of the article is organized as follows. Section 2 presents the method of the principal component analysis. Section 3 applies it to crude oil futures prices' curves for different periods and maturities. Section 4 proposes a comparison with previous studies on the term structures of interest rates. Section 5 concludes.

II Principal component analysis: the method[1]

Principal component analysis can be used as a descriptive statistical method involving a mathematical procedure that reduces the dimensionality of a data set by collapsing the information it contains. In a system including a large number of observed variables, especially when there is significant correlation among the variables that are being described, groups of variables often evolve in unison because they are influenced by the same driving forces. Usually, the analysis shows that there are only a few of such driving forces, and the method offers a tool which fulfills two contradictory goals: on the one hand, the wish to simplify some problems by reducing the dimension of the representation; on the other hand, the wish to preserve as much as possible of the original information content.

The reduction of the dimensionality of the data set is obtained by transforming the initial matrix (n observations $\times N$ variables) in a reduced matrix containing M factors and n observations: Factors ($j = 1,....,M$)

$$\text{Observations } (i = 1,\ldots, n) \begin{bmatrix} f_{1l} & f_{1M} \\ & f_{ij} \\ f_{n1} & f_{nM} \end{bmatrix}$$

where f_{ij} is the value of the factor j for the observation i and $M < N$. The problem is simplified by replacing a group of variables with less new variables, which are uncorrelated and called principal components.

The principal components can be considered as sources of risk or information, and they summarize the main features of the original variables. Each principal component is a linear combination of the original variables:

$$F_j = \sum_{k=1}^{N} a_{jk} f_k$$

where F is a factor, X is the original variable, and a is a cœfficient.

The extraction of principal components amounts to a variance maximizing rotation of the original variables space (when there are more than two variables in a system, it is possible to define them as a space, instead of a plane). With more than three variables, the space has more than three dimensions, and it becomes very difficult to simultaneously envision all these variables. The principal component analysis takes the cloud of data points, and rotates it such that the maximum variability is visible.

It is possible to consider the first principal component axis as a regression line that represents the "best" summary of the linear relationship between the variables. This regression line is rotated so that it maximizes the variance (variability) of the first new variable, namely the first component. In other words, the first principal component is the projection on the direction in which the variance of the projection is maximized. Meanwhile, the first component minimizes the distance of each point of the cloud to that line. Thus, this axis is as close to all of the data as possible. Equivalently, the line gœs through the maximum variation in the data.

After this first axis of maximal variance has been extracted, there remains some variability around this line. It is then is possible to determine another line that maximizes the remaining variability, and so on. Having determined the first (k – 1) components, the k^{th} component is always determined as the principal component of the residuals. In this manner, consecutive factors are extracted. They are ordered in the order of descending variability in the data.

Because each consecutive factor is defined to maximize the variability that is not captured by the preceding factor, consecutive factors are independent of each other. In other words, consecutive factors and axes are completely mutually uncorrelated, or orthogonal to each other:

$$\rho(F_j, F_m) = 0 \qquad j \neq m$$

where ρ is the correlation cœfficient.

There are as many principal components as original variables and, taken together, they explain all the variability in the original data. However, the sum of the variances of the first principal components usually exceeds 80% of the total variance of original data. Examining these few first factors thus authorizes a deeper understand of the driving forces that influence the original data set. Indeed, a projection into a subspace of a very low dimension is useful for visualizing the data. Naturally, the choice of fewer components than the number of original variables leaves in the system some residual variability that remains unexplored. However, the data not contained in the first components may be mostly due to noise.

The question of how many factors to extract is left unanswered, on a theoretical point of view. As consecutive factors are extracted, they account for less and less variability. The decision of when to stop extracting factors depends on when there is only very little variability left. The nature of the decision is somehow arbitrary. There are several methods to help in this choice of the number of components. The most obvious consists in calculating the contribution of each component to the total variability of the system, as will be done in the following study. Indeed, the principal component analysis may reveal the importance of each principal component (its "factor score", which is an expression of the contribution of that source of risk or information to the variance of the initial variables). An additional important aspect, as we will see, is the extent to which a solution is interpretable.

III Principal component analysis of crude oil prices curves

The aim of this study is twofold. Firstly, it intends examining the possible structural nature of the crude oil futures prices movements. Secondly, it investigates whether there is an influence of the maturity on this dynamic. Thus, the study examines the prices' behavior on a long period of time, from June 1989 to January 2002, and it includes long-term futures contracts.

1. *Data*

The database is an important element of the study. In 2005, the most developed commodity futures market, considering the volume and maturity of the transactions, is the American crude oil futures market. Working with crude oil futures prices makes it possible to study maturities as far as seven years[2]. The data are daily settlement prices for the West Texas Intermediate futures contract traded at the New York Mercantile Exchange (Nymex). They have been operated such as the first futures price's maturity corresponds to the one month maturity, such as the second futures price corresponds to the two months maturity, and so forth. As a result of the evolution process of the market, new contracts with longer maturities were introduced during the period. Delivery dates were indeed progressively extended from 15 to 84 months between 1989 and 1999. Consequently, the information relative to long-term contracts is only available on the period 1999-2002.

2. *Principal components*

The first part of the study aims identifying the dynamic prices' behavior on the whole period, from the 06/06/1989 to the 01/14/2002. It includes maturities from the first to the 15th months (15 x 163 futures prices). When applying the principal component analysis, there are initially as many original variables as different futures prices of various expiry dates. Thus *Table 1*, which presents the factors obtained on the whole study period, contains 15 factors, or components.

Table 1. **Principal components, 1989-2002.**

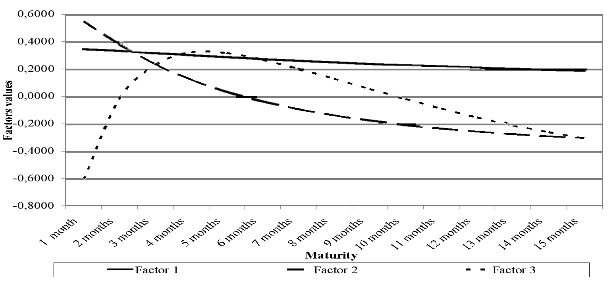
	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7	Factor 8	Factor 9	Factor 10	Factor 11	Factor 12	Factor 13	Factor 14	Factor 15
1 month	0.343	0.5475	-0.6118	-0.4062	0.2022	-0.0432	0.0047	0.0219	-0.007	0.0081	-0.0048	0.003	-0.0003	-0.0004	0.0001
2 months	0.3296	0.378	-0.0206	0.4821	-0.5718	0.3968	-0.0014	-0.1749	0.0236	-0.0055	0.0051	-0.0051	0,0000	0.002	-0.0002
3 months	0.314	0.2352	0.2269	0.3604	0.0499	-0.5919	-0.0128	0.5414	-0.1303	0.035	-0.0209	0.0075	-0.0009	-0.0022	-0.0002
4 months	0.2983	0.1207	0.3197	0.0968	0.379	-0.1825	0.0226	-0.5232	0.4459	-0.2667	0.2233	-0.1005	0.057	-0.0234	-0.0015
5 months	0.2837	0.0325	0.3148	-0.0548	0.2933	0.1679	-0.0084	-0.2356	-0.2942	0.3413	-0.528	0.3161	-0.2437	0.0731	0.0069
6 months	0.2699	-0.038	0.2661	-0.1736	0.1526	0.3258	-0.0472	0.1199	-0.3787	0.1807	0.3109	-0.3807	0.5048	-0.0782	-0.0097
7 months	0.2573	-0.0944	0.1988	-0.2407	-0.0137	0.2924	-0.103	0.3237	0.0209	-0.3019	0.3601	0.0879	-0.6288	0.0373	0.0258
8 months	0.2458	-0.1403	0.1282	-0.2584	-0.1553	0.1373	-0.1383	0.3009	0.4371	-0.268	-0.4405	0.2115	0.4215	-0.0512	-0.0465
9 months	0.2351	-0.1782	0.0533	-0.24	-0.2864	-0.1714	0.853	-0.0789	-0.0787	-0.0301	0.0096	0.0059	-0.0043	-0.0022	-0.0038
10 months	0.2251	-0.2105	-0.0196	-0.1772	-0.2596	-0.1887	-0.2346	-0.0523	0.3101	0.475	-0.0715	-0.4625	-0.1823	0.3429	0.1442
11 months	0.2159	-0.2373	-0.0903	-0.0916	-0.2146	-0.2272	-0.265	-0.1797	-0.0298	0.2515	0.2164	0.2737	-0.0294	-0.6202	-0.3301
12 months	0.2074	-0.2594	-0.1559	0.0123	-0.1112	-0.1799	-0.2246	-0.1962	-0.2949	-0.2007	0.1912	0.383	0.2148	0.3352	0.5125
13 months	0.1993	-0.2777	-0.2137	0.1284	0.0317	-0.0638	-0.1146	-0.1136	-0.288	-0.389	-0.199	-0.2651	-0.0458	0.2765	-0.6059
14 months	0.1918	-0.2932	-0.2637	0.248	0.1851	0.0916	0.0537	0.0412	-0.0298	-0.1587	-0.2717	-0.3403	-0.1435	-0.4989	0.4624
15 months	0.1847	-0.3061	-0.308	0.3606	0.3306	0.2386	0.2189	0.2063	0.2966	0.33	0.2203	0.2662	0.0811	0.2098	-0.1539

In highly correlated data, such as crude oil futures prices, typically only two or three of them should be significant, accounting for nearly all the variation or movement in the data set. Moreover, the three first factors are the more easy to interpret. They are illustrated by *Figure 1*.

The first factor corresponds to a roughly parallel shift in the prices curve: all of this factor's values are of the same sign (positive) and they are of roughly the same magnitude. Therefore, a shock in the futures prices via this first factor affects the entire curve in a uniform manner, resulting in a parallel move of the prices curve. Whatever the maturity considered, one unit of that factor corresponds to an increase of the futures price. The increase is for example around 30 cents for the four months maturity, and 21 cents for the 12 months maturity. The weights decrease with the maturity. Thus, the stronger impact is associated with the nearest futures price. This phenomenon is due to the fact that futures prices have different volatilities. Indeed, one of the most important features of the commodity prices curve's dynamic is the difference between the price behaviour of first nearby and deferred contracts. The movements in the prices of the prompt contracts are large and erratic, while the prices of long-term contracts are relatively still. This results in a decreasing pattern of volatilities along the prices curve. This phenomenon is usually called "the Samuelson effect". Intuitively, it happens because a shock affecting the nearby contract price has an impact on succeeding prices that decreases as maturity increases (Samuelson, 1965). As futures contracts reach their expiration date, they react much stronger to information shocks, due to the ultimate convergence of futures prices to spot prices upon maturity. These price disturbances, influencing mostly the short-term part of the curve, are due to the physical market, and to demand and supply shocks.

The Samuelson effect can be eliminated by a standardization of the futures prices. In that case, the same role is attributed to each maturity in the definition of the proximity between two observations. *Table 1A, in the Appendix*, illustrates the results obtained with standardized prices. The most dramatic change concerns the first factor, which appears more clearly as a level factor, because factor's

Figure 1. **The three factors driving the futures prices curve movements**



values are then really close to each other. In that case, it is possible to adhere to a strict definition of a parallel shift.

Although the principal component analysis dœs not give an interpretation of the factors, and dœs not explain what actually drives this futures prices changes, the explanation of such parallel moves of the entire futures prices may lie in variations of economic variables, such as production and consumption, or expectations of exhausting supply, improving technology for the production and discovery of the commodity. They could also be due to elements that are exogenous to the physical market, such as expected inflation, interest rates, prices for renewable energies, as well as political and regulatory effects.

The second factor corresponds to a "steepening" or a "twist" of the prices curve: when the nearest futures prices move in one direction, the deferred prices move in the other one. More precisely, one unit of the second factor corresponds to an increase of the shorter maturities, and simultaneously to a decrease of the longer maturities. The inflection point is located at the 6th month, and the stronger impact is associated with the two extremities of the prices curve: it is attributed to the nearest futures price, which is followed by the 2nd and the 15th months.

A change in the slope of the futures prices curve can be explained by a change in the level of stocks, the fear of inventory disruptions or, more generally, a change in the market risk premiums.

The third factor corresponds to a curvature of the term structure. Indeed, futures prices of the shorter and longer maturities move in the same direction, whereas middle maturities (3rd to 9th months) move in another one. The curvature of the curve could be related to the volatility of futures prices and to the quality of the shocks transmission along the prices curve, via the Samuelson effect.

These three factors are the most important because they describe virtually all possible futures prices curves. A factor's importance can be measured by the standard deviation of its factor's score, which is an expression of the contribution of that source of risk to the volatility of the futures prices. Because there are as many maturities as factors in the test presented here, the futures prices changes observed on any given day can always be expressed as a linear sum of the factors, by solving a set of N simultaneous equations, where N is the number of maturities. The amounts of the factors in the prices moves, on a particular day, are known as the factor or component scores.

The standard deviations of the factors scores are shown in *Table 2*. From the variances of the factors, it is easy to calculate the total variability explained by each principal component. In the case presented here, the total variance of the factors is 198.24. The first factors explains $((13.9091)^2/198.24) = 97.59\%$ of this total variance. Thus, the first two factors account for 99.86% of the variance, the third factor accounts only for 0.10%. Therefore, most of the risk associated with futures prices move is accounted for one or two factors, instead of all 15 futures prices, and all the other factors can be neglected.

Table 2. **Standard deviation and variability explained by each component, 1989-2002**

	Stand. Dev.	%
Factor 1	13,9091	97,5901
Factor 2	2,1223	2,2720
Factor 3	0,4506	0,1024
Factor 4	0,2317	0,0271
Factor 5	0,1015	0,0052
Factor 6	0,0566	0,0016
Factor 7	0,0412	0,0009
Factor 8	0,0316	0,0005
Factor 9	0,0173	0,0002
Factor 10	0,0141	0,0001
Factor 11	0,0100	0,0001
Factor 12	0,0000	0,0000
Factor 13	0,0000	0,0000
Factor 14	0,0000	0,0000
Factor 15	0,0000	0,0000

Some of our results are thus in line with previous studies on interest rates. Indeed, these studies also lead to the identification of the level, the steepening and the curvature factors, that are mutually hortogonal. What is more specific of commodity prices is that the third factor can actually be neglected, and the second is not really important, as far as these maturities are concerned. Relying on that basis, it is possible to study the eventual structural nature of prices movements, and the impact of the maturity on prices behavior.

3. *The permanent nature of principal components*

In order to examine the structural nature of prices movements, three different studies are compared. The first corresponds to a long period (06/06/1989-01/14/2002), the second is centred on a short one (06/01/1999-01/14/2002), and the third is dedicated to a specific period, during which prices where especially volatile and backwardation was particularly strong: the 1st Gulf War (06/01/1990-02/01/1991). The maturities retained for these studies are similar: respectively 15, 18 and 17 months[3].

The examination of the principal components obtained on the two new periods, illustrated by *Tables 3 and 4*, leads to the identification of the same three factors that were extracted from *Table 1*.

Table 3. **Principal components, 1999-2002**

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7	Factor 8	Factor 9	Factor 10	Factor 11	Factor 12	Factor 13	Factor 14	Factor 15	Factor 16	Factor 17	Factor 18
1 month	0.3016	0.5222	-0.6945	-0.2769	0.2498	0.1113	-0.0492	-0.013	0.0012	-0.0075	0,0000	0.0009	-0.0005	0.0028	0.0004	-0.0014	-0.0007	0.0014
2 months	0.2855	0.3861	-0.0314	0.3569	-0.5801	-0.4482	0.2809	0.1226	0.0601	0.0645	0.0415	-0.0168	0.0015	-0.0015	-0.0025	0.0034	0.0039	-0.0032
3 months	0.275	0.2722	0.2387	0.3681	-0.0762	0.2863	-0.4798	-0.3522	-0.3087	-0.2583	-0.2121	0.0831	0.0028	-0.0392	0.0125	-0.0074	-0.0124	0.0004
4 months	0.267	0.1803	0.3014	0.1964	0.2093	0.349	-0.0461	0.1191	0.2969	0.454	0.4457	-0.2414	0.0454	0.1741	-0.0175	0.018	0.0238	-0.006
5 months	0.26	0.1049	0.2836	0.0156	0.2982	0.0991	0.2866	0.2986	0.282	-0.1066	-0.3488	0.4028	-0.1855	-0.3996	0.0027	0.0099	-0.0414	0.0099
6 months	0.2533	0.0416	0.2486	-0.1204	0.2778	-0.1572	0.2843	0.094	-0.1386	-0.4412	-0.1168	-0.2367	0.2041	0.5682	0.0145	-0.074	0.0955	0.0582
7 months	0.2468	-0.011	0.2028	-0.2236	0.185	-0.2738	0.0959	-0.1322	-0.44	0.0227	0.3476	-0.2058	0.0946	-0.5217	-0.0962	0.0152	-0.2002	-0.1544
8 months	0.2404	-0.0544	0.1513	-0.2784	0.0365	-0.279	-0.0966	-0.2325	-0.1547	0.3751	-0.0306	0.2739	-0.4306	0.2299	0.3091	0.1537	0.2631	0.1704
9 months	0.2341	-0.0925	0.0986	-0.2923	-0.1103	-0.1832	-0.2374	-0.1839	0.2925	0.2369	-0.2748	0.1082	0.2257	0.1519	-0.4549	-0.2906	-0.274	-0.1901
10 months	0.228	-0.1264	0.0472	-0.2636	-0.2101	-0.0311	-0.2561	0.0132	0.3585	-0.2045	0.0094	-0.1721	0.3676	-0.2616	0.1316	0.3717	0.3283	0.2873
11 months	0.222	-0.1566	-0.0014	-0.2053	-0.2574	0.113	-0.1874	0.2246	0.1358	-0.259	0.1172	-0.1673	-0.3398	0.031	0.4565	-0.244	-0.2672	-0.3694
12 months	0.216	-0.1836	-0.0463	-0.131	-0.2516	0.234	-0.0385	0.3283	-0.2448	-0.1238	0.2178	0.1026	-0.2883	0.0498	-0.5233	-0.0866	0.0452	0.4133
13 months	0.2098	-0.2069	-0.0856	-0.0428	-0.187	0.3017	0.1721	0.1853	-0.3134	0.1761	-0.0708	0.2822	0.3396	0.1204	0.0327	0.3402	0.1328	-0.4857
14 months	0.2034	-0.2268	-0.1169	0.0447	-0.1017	0.2446	0.2825	-0.1291	-0.1408	0.2693	-0.2539	-0.1485	0.2299	-0.0915	0.3407	-0.3128	-0.2526	0.4551
15 months	0.1971	-0.2437	-0.1444	0.1239	-0.0126	0.1343	0.2879	-0.3882	0.1436	-0.0263	-0.1771	-0.3928	-0.346	-0.1142	-0.243	-0.0154	0.3905	-0.2497
16 months	0.1911	-0.2583	-0.1655	0.2001	0.0862	-0.034	0.1431	-0.3395	0.2214	-0.2254	0.2663	0.2331	-0.0592	0.1619	-0.0259	0.4195	-0.4938	0.0987
17 months	0.1853	-0.2708	-0.1871	0.2785	0.1837	-0.1783	-0.0999	-0.0245	0.0649	-0.1119	0.3267	0.367	0.2139	-0.0592	0.0993	-0.5002	0.3659	-0.048
18 months	0.18	-0.2818	-0.2106	0.3587	0.278	-0.3029	-0.3515	0.4089	-0.111	0.1668	-0.287	-0.272	-0.0754	-0.002	-0.0369	0.2011	-0.1062	0.0116

In *Table 3 and 4*, the parallel shifts (Factor 1) present the same characteristics as those described previously: the weights are all positive, and the more important are attributed to the short part of the curve. This is especially true for the 1st Gulf War period, where the volatility of the nearest contracts is particularly high.

Table 4. **Principal components, 1990-1991**

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7	Factor 8	Factor 9	Factor 10	Factor 11	Factor 12	Factor 13	Factor 14	Factor 15	Factor 16	Factor 17
1 month	0.4128	-0.5231	0.6755	0.3122	0.0261	0.0089	-0.0372	-0.0005	0.0089	-0.0058	-0.002	0.0002	-0.0014	-0.0005	0.0001	-0.0001	0
2 months	0.3838	-0.3438	-0.214	-0.6442	0.4755	-0.2006	0.0854	-0.0032	0.0013	0.0029	0.0023	-0.0088	0.0012	0.0032	0	0.0004	0.0003
3 months	0.3445	-0.1907	-0.2415	-0.2565	-0.6043	0.4891	-0.3411	0.0001	0.0003	0.0046	-0.0006	0.0053	-0.0031	-0.0038	-0.0011	0.0009	0.0014
4 months	0.3097	-0.0566	-0.2728	0.2121	-0.3427	-0.223	0.6659	-0.1926	-0.3235	0.1457	0.0451	-0.0714	-0.0225	0.0009	0.0175	-0.0001	-0.008
5 months	0.2802	0.0307	-0.2696	0.2578	-0.0887	-0.2825	0.0153	0.2069	0.5621	-0.4001	-0.1787	0.3462	0.1408	-0.0252	-0.0483	-0.0054	0.0072
6 months	0.2543	0.0947	-0.2266	0.2756	0.0876	-0.2222	-0.3222	0.3062	0.0828	0.1788	0.2383	-0.5965	-0.2889	0.0727	0.0652	-0.0043	-0.0031
7 months	0.2323	0.1399	-0.1617	0.2391	0.2124	-0.0509	-0.3468	0.0346	-0.3585	0.445	0.0465	0.4719	0.3247	-0.0936	-0.091	-0.0022	0.0047
8 months	0.2131	0.1716	-0.0999	0.176	0.236	0.1381	-0.1689	-0.3205	-0.3419	-0.4081	-0.4721	-0.0334	-0.2609	0.2559	0.1862	0.0499	0.0064
9 months	0.1965	0.1924	-0.0332	0.1069	0.2097	0.2584	0.0309	-0.3988	0.1095	-0.23	0.2493	-0.2536	0.1799	-0.5627	-0.308	-0.0765	0.0101
10 months	0.1822	0.2085	0.0291	0.0395	0.1645	0.3122	0.1681	-0.2267	0.3291	0.1223	0.4244	0.1054	0.1241	0.5626	0.2443	0.0919	-0.0464
11 months	0.1705	0.2225	0.0811	-0.0231	0.1122	0.2771	0.2235	0.1423	0.2357	0.3642	-0.2396	0.2139	-0.5361	-0.3704	0.1923	-0.0321	0.0286
12 months	0.1608	0.2325	0.119	-0.0656	0.0502	0.2021	0.1938	0.2967	0.0079	0.1256	-0.358	-0.2037	0.17	0.3264	-0.6207	-0.1587	-0.0352
13 months	0.1527	0.2411	0.1491	-0.1036	-0.0056	0.0945	0.1313	0.3895	-0.155	-0.1396	-0.0809	-0.1945	0.4292	-0.1831	0.401	0.4589	0.1752
14 months	0.1458	0.2492	0.1722	-0.135	-0.0586	-0.039	0.0293	0.2579	-0.2324	-0.2749	0.2388	0.1054	-0.0079	-0.0434	0.2118	-0.5783	-0.4664
15 months	0.1395	0.256	0.1925	-0.1606	-0.1143	-0.1572	-0.0357	0.0529	-0.1538	-0.1982	0.319	0.2068	-0.2811	0.0851	-0.219	-0.0248	0.6841
16 months	0.1337	0.2621	0.2093	-0.1831	-0.1648	-0.2679	-0.1109	-0.1481	0.0075	0.0013	0.0637	0.0865	-0.2008	-0.0155	-0.2589	0.5714	-0.5072
17 months	0.1283	0.2668	0.2227	-0.2003	-0.2107	-0.3621	-0.1741	-0.401	0.2223	0.2673	-0.295	-0.183	0.2348	-0.0078	0.2294	-0.2913	0.1484

Tables 3 and 4 give also evidence of the presence of a second factor, leading prices to move in different directions according to their maturities. The inflection point is located around respectively the 7th and 5th months, and the most important weights are once again attributed to the extremities of the curve. The only difference in the results obtained concerns the signs of the second factor for the 1st Gulf War period. In that case, the second factor is negative for the nearest maturities, and then positive, whereas it presents the opposite profile on the other periods. Lastly, a third factor can also be identified. The two inflection points are located around the second and the 10th months, and, one more time, the period of the 1st Gulf War presents opposite signs.

Thus, whatever the period taken into account, three main components can be identified, that characterize the futures prices movements. Moreover, *Tables 1, 3 and 4* show that the factors values evolve in the same intervals. This structural nature of prices behavior is even more pronounced if we consider the total variability explained by each component (*Tables 5 and 6*).

Table 5. **Standard deviation and variances of the components, 1999-2002**

	Stand. Dev.	%
Factor 1	13,7166	95,4689
Factor 2	2,9267	4,3464
Factor 3	0,5394	0,1476
Factor 4	0,2373	0,0286
Factor 5	0,1072	0,0058
Factor 6	0,0548	0,0015
Factor 7	0,0361	0,0007
Factor 8	0,0224	0,0003
Factor 9	0,0141	0,0001
Factor 10	0,0141	0,0001
Factor 11	0,0100	0,0001
Factor 12	0,0100	0,0001
Factor 13	0,0000	0,0000
Factor 14	0,0000	0,0000
Factor 15	0,0000	0,0000
Factor 16	0,0000	0,0000
Factor 17	0,0000	0,0000
Factor 18	0,0000	0,0000

For all the periods considered, the relative importance of the three factors is similar: the first factor explains at least

95% of the total variance, and gathering the first and second factors accounts for at least 99.7% of this variance. Thus, it appears clearly that structurally, the first factor has the stronger impact, the second factor a lower one, and that the third factor can be neglected.

Table 6. **Standard deviation and variances of the components, 1990-1991**

	Stand. Dev.	%
Factor 1	16,4114	97,5787
Factor 2	2,4257	2,1317
Factor 3	0,6920	0,1735
Factor 4	0,5082	0,0936
Factor 5	0,1942	0,0137
Factor 6	0,1241	0,0056
Factor 7	0,0854	0,0026
Factor 8	0,0316	0,0004
Factor 9	0,0173	0,0001
Factor 10	0,0141	0,0001
Factor 11	0,0100	0,0000
Factor 12	0,0100	0,0000
Factor 13	0,0000	0,0000
Factor 14	0,0000	0,0000
Factor 15	0,0000	0,0000
Factor 16	0,0000	0,0000
Factor 17	0,0000	0,0000

This permanent nature of prices movements is important, because it can be exploited for modelling purposes, as we will see it in paragraph 5 of this section. Our result show, indeed, that only two factors can be taken into account in order to explain the prices behaviour, at least for the shorter maturities (up to 18 months). These two factors are the parallel shift, that stands for upward or downwards movements of the entire curve, and the steepening, that represents changes in the slope of the curve.

Another preoccupation appears however when reaching modelling purposes. Indeed, a model is presumed to be able to represent prices behaviour for every maturity. Until now, however, this study was concentrated on rather short-term delivery dates. Do the results of the principal component analysis change when long-term futures contracts are taken into account? Answering this question constitutes the objective of the next part of this article.

4. *The impact of maturity on prices behavior*

The analysis of the impact of maturity on prices behaviour is obtained by comparing the results found on the same period – 1999-2002 – but on different maturities – 1st to 18th months in the first case, 1st to 84th months in the second case. *Tables 7 and 8* present, respectively, the principal components and their variance for the long-term maturities[4].

Table 7. **Principal components, 1999-2002, 1 to 84 months**

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7	Factor 8	Factor 9	Factor 10	Factor 11	Factor 12	Factor 13	Factor 14
1 month	0.4081	0.5496	0.4658	-0.4905	0.2368	-0.1133	0.0381	0.0426	-0.0397	0.0031	-0.0114	0.0064	-0.003	0.0011
3 months	0.3766	0.3314	0.1242	0.433	-0.4701	0.3902	-0.2205	-0.2755	0.0967	-0.1386	0.1183	0.0332	0.0617	-0.015
6 months	0.3523	0.1371	-0.1484	0.3851	-0.1083	-0.1981	0.2062	0.4289	-0.2724	0.3191	-0.3652	-0.1451	-0.2672	0.0868
9 months	0.3291	0.02	-0.2599	0.2083	0.1812	-0.3295	0.2179	0.1063	0.263	0.048	0.34	0.1545	0.5777	-0.1886
12 months	0.3066	-0.0654	-0.2671	0.0628	0.3044	-0.1385	0.0126	-0.2045	0.3157	-0.3143	0.1868	0.0069	-0.5835	0.3144
15 months	0.2821	-0.1261	-0.2492	-0.0533	0.2624	0.0683	-0.1852	-0.2848	-0.1561	-0.2367	-0.4866	-0.1906	0.0819	-0.5389
18 months	0.2596	-0.1684	-0.2101	-0.1412	0.1439	0.1782	-0.2774	-0.1283	-0.3376	0.2097	-0.1069	0.3632	0.2837	0.5591
21 months	0.2409	-0.2003	-0.1424	-0.2279	-0.0456	0.1932	-0.2741	0.1944	-0.2178	0.2988	0.5829	-0.316	-0.1879	-0.2561
24 months	0.2253	-0.2232	-0.0908	-0.3097	-0.2634	0.3621	0.7588	-0.1253	-0.0156	-0.0456	-0.0258	0.0136	0.0015	0.0091
28 months	0.205	-0.2463	-0.006	-0.284	-0.3026	0.0335	-0.2986	0.4802	0.5488	-0.1097	-0.287	0.0722	0.049	0.0317
48 months	0.1604	-0.308	0.221	-0.0452	-0.3876	-0.5312	-0.0511	-0.0717	-0.3952	-0.3331	0.1179	0.2965	-0.0899	-0.1066
60 months	0.1377	-0.3092	0.318	0.0573	-0.0522	-0.2402	-0.0056	-0.2914	0.1246	0.1376	-0.0593	-0.6627	0.2379	0.3193
72 months	0.1149	-0.3004	0.373	0.1501	0.1515	0.0331	0.0129	-0.2377	0.2512	0.5503	-0.0847	0.3915	-0.2342	-0.2698
84 months	0.0973	-0.2985	0.4277	0.3117	0.3965	0.3479	0.0751	0.3963	-0.1551	-0.3764	0.0802	-0.0048	0.0785	0.0457

Table 7 authorizes the identification of the three factors describing the prices movements. The comparison with *Table 3* shows that the values of the first factor are less homogeneous when the whole price curve is taken into account. Thus, the shifts of the curve are a less parallel than before, because the differences in the volatilities of the futures prices increase with maturity. As in *Table 3*, the most important weight is associated to the short-term extremity of the curve. As far as the second factor is concerned, the steepening seems to be a bit more pronounced with longer prices curves. Lastly, the third factor is here totally different: its values are positive for the two ends of the curve, and negative for the intermediate maturities. *Table 3* gives evidence of an opposite profile for this factor.

This difference in the third factor is not really important, as *Table 8* illustrates it: indeed, the third factor accounts for 0.52% of the total variance of the futures prices… Thus, even if increasing the maturity gives more weight to the curvature factor (with the short-term maturities, its level was 0.15%), this weight is still marginal. What is more important in Table 8 is that the relative importance of the two first factors varies depending on the maturity. Now that the whole prices curve is taken into account, the second factor explains more than 10.8% of total variability, whereas its level was 4.35% with short-term contracts.

Table 8. **Standard deviation and variances of the components, 1999-2002, 1-84 months**

	Stand. Dev.	%
Factor 1	9,7682	88,3543
Factor 2	3,4174	10,8143
Factor 3	0,7521	0,5238
Factor 4	0,4703	0,2048
Factor 5	0,2711	0,0681
Factor 6	0,1257	0,0146
Factor 7	0,1010	0,0094
Factor 8	0,0632	0,0037
Factor 9	0,0592	0,0032
Factor 10	0,0490	0,0022
Factor 11	0,0283	0,0007
Factor 12	0,0173	0,0003
Factor 13	0,0173	0,0003
Factor 14	0,0141	0,0002

Comparing the results of principal component analysis on different periods and maturities gives thus key insights about the number and shape of the underlying factors driving the dynamics of the term structure of commodity prices. Our results show that for short-term analysis, one factor can be retained. However, when the maturity of the contracts increases, it becomes important to integrate a second factor. Lastly, there is no real need of a third factor, because its importance is marginal, even with seven years futures contracts.

5. *The use of principal component analysis for hedging purposes*

There are several ways to use this principal component analysis for hedging purposes.

The first way consists in treating the principal components as the two (or three) factors that govern the movements of all futures prices, and determine the sensitivity of various portfolios to movements in each of the factors. Portfolios of different maturities will indeed have different exposures to the different sources of risk. Short maturity positions, for instance, will be mostly sensitive to the parallel shift of the price curve, while long term instruments will be sensitive to the two (or three) types of movements. In order to measure this sensitivity, it is possible to associate each principal component with the standard deviations of the factors score. This association will define how the price of each futures contract composing the portfolio will change in response to a shock to the component. For example, gathering *Tables 7 and 8* shows that a one standard deviation move in the first factor corresponds to the 6 months futures prices moving by 9.7682 x 0.3523 = 3.44 US dollar, the 48 months futures prices moving by 9.7682 x 0.1604 = 1.57 US dollar and so on.

Principal component analysis gives also a way to determine the Value At Risk of a portfolio whose primary source of risk is that associated with the term structure of futures prices. Var analysis is then based on the probability of a change in the value of a portfolio attributable to a change in the factors. Using the variances of the factors, it is possible to determine the future distribution of the futures prices curve, using for example Monte Carlo simulations. This distribution can be then be used in order to generate a price distribution of each position of the portfolio, as well as the distribution of the value of the entire portfolio. The Value at Risk for any confidence level is then observed directly from the distribution of values[5].

Principal component analysis, when it is used as a purely descriptive approach, can also be exploited as an empirical basis that supports the theoretical analysis developed for futures prices models. Term structure models of commodity prices borrow to the contingent claim analysis developed for options and interest rates models. Their objective is to reproduce the futures prices observed in the market as accurately as possible. They also provide a mean for the discovery of futures prices for horizon exceeding exchange-traded maturities. These models are developed in a partial equilibrium framework. Thus, the state variables (or factors) are supposed to be exogenous in these models, and the choice of their nature and number can be regarded as somehow arbitrary. With a principal component analysis, it is possible to observe that only two factors are sufficient to explain more than 99% of prices' volatility, even when the whole curve is taken into account. Consequently, it is relevant to retain solely two state variables in a term structure model. Moreover, if the steepness factor has a significant impact for long-term maturities, then a long-term analysis should retain a mean-reverting behavior for at least one of the state variables of the model.

IV Comparison with interest rates

A comparison with previous studies performed in the case of interest rates shows that our results are comparable to those obtained in that field. Indeed, these studies lead, most of the time, to the identification of three factors: level, steepness and curvature factors. The interpretation of these factors is naturally different. For interest rates, a parallel movement of the entire curve may be caused primarily by changes in expected inflation. A change in the slope of the curve could be due to by changes in expected long term inflation or a change in the market risk premiums. Lastly, the curvature of the yield curve is usually related to the volatility of interest rates.

The seminal work on principal components of the term structure of interest rates is due to Litterman and Scheinkman (1991). These authors found that three factors, or attributes, explain 98% of the variation in yields associated with Treasury base securities, for 1 year to 18 years maturity. The first factor represents approximately a parallel change in yields. It accounts for 89.5 percent of the total explained variation. The second factor lowers the yield of zerœs up to 5 years and raises the yields for longer maturities. It explains 8.5 percent of the total variance. The effect of the third factor in Treasury zerœs is to increase the curvature of yield curve in the range of maturities below 20 years. It accounts for 2.0 percent of the variance. In average, these three factors explain 98.4 percent of the total variability of yields. Thus, as observed with crude oil futures prices, the first factor is by far the most important, explaining thus the success of sensitivity indicators such as the Macaulay duration. However, as the maturities are much longer in interest rates than in crude oil markets, the inflection point of the second factor changes with the underlying asset. One the one hand, it is situated around the 5th year, and on the other hand, it is located around the 6th month. Lastly, if the third factor can be neglected in the case of crude oil prices, this is not possible anymore for interest rates.

In 1997, Frye confirms these results with even longer maturities. Working with interest rates having for one month to 30 years maturity, he shows that the first factor accounts for 83 percent of the total variability, the second for 10 percent, and the third for 2.8 percent. These three factors explain almost 96 percent of the total variability of interest rates. Thus, as was the case with crude oil futures prices, it seems that the extension of the curve to longer maturities leads to a rebalancing of the relative importance of the three factors in the favour of the second and third factors[6]. This corroborates the assumption that, if the horizon of futures transaction further increases in the crude oil market, the third factor will become more important for long-term exposures.

The work of Knez, Litterman and Scheinkman (1994) is a bit different because it offers a distinction between interest rate and credit risks. Indeed, these authors identify four common factors that describe money market returns. The first is called a level factor and represents, on average, 73 percent of the total explained variation. The second is called steepness and it accounts for 12 percents of the total explained variation in returns. The third factor is referred to as a Treasury factor. It involves movements in the yield curve characterized by the private issuer money market instruments moving uniformly away from the Treasury bill market and captures the credit risk in Treasury issues. It accounts for approximately 15 percent of the total variation. This credit risk in the private issuer instruments combines bank risk and firm risk. To disentangle between these two elements, the authors expand the set of instruments to include higher credit risks and examine a fourth factor, which is referred to as a private issuer factor. It represents movements in the yield curve characterized by commercial paper instruments moving uniformly away from the other private sector instruments, and it accounts for 4 percent of the total variation.

This decomposition between different sources of risk could be transposed into commodity markets. Indeed, provided that the information needed is available, the method could be used in order to take into account and to quantify the risk associated with price differentials. The latter appear, for example, when commodities are characterized by different levels of quality, especially when the quality of a specific commodity is not the same as the one corresponding to the underlying product of the futures contract. Prices differentials are very important for commodity markets, because standardization is more difficult than it is for many financial instruments. As far as the petroleum market is concerned, price differentials are for example crucial in the case of jet fuel. Operators who need to protect themselves against the prices fluctuations of this product usually resort to the so called "dirty hedging" or cross hedging techniques: they use futures contracts on crude oil in order to hedge their physical positions in jet fuel. Consequently, the price differential between crude oil and jet fuel, and more precisely the stability of the spread between the two prices, is very important for them.

Several other studies based on principal components or factor analyses were realized in the field of interest rates. For example, Fung and Hsieh (1996) examined the estimation errors associated with this kind of analyses, during major yield curve events, due to the linear approximations on which they rely. Perignon and Villa (2004) show that there is a need to take into account the time-varying nature of the covariance of interest rates and propose a novel method of extracting the risk factors driving interest rates. This kind of analyses draws avenues for future researches in the case of commodity markets.

V Conclusion

Applying a principal component analysis to crude oil futures prices' curves leads to the identification of the type of prices curves movements. Three different kinds of movements can be distinguished: a parallel shift in the curve (first factor), a steepening of the curve (second factor) and the curvature (third factor). Moreover, the principal component analysis makes it possible to calculate the contribution of each component to volatility. This calculus shows firstly, that when the prices curves are shortened, the importance of the first factor increases dramatically and secondly, that the third factor can always be neglected.

This factor decomposition is particularly useful because it provides a parsimonious representation of the term structure of futures prices. However, it is particularly of use when variables are linearly related to each other. The study of non linearity or heteroscedasticity in the commodity futures prices and their impact on factor decomposition is left for further study. ■

1 For more detail on this method, see for example Jackson (1991).

2 However, the transaction volume is not very high for the longer maturities. Therefore, the quality of the information conveyed by those prices may not be excellent.

3 We retained a 15 months maturity for the period 1989-2002 because it was the farthest maturity for a futures contract in 1989. In 1990-1991, the farthest maturity was 17 months. Lastly, we retained only the 18 first months for the period 1999-2002, in order to make comparisons (see paragraph 4) between short- and long-term futures contracts.

4 For this part of the study, 32 different maturities were available, from the 1st to the 28th months, and the 48th, the 60th, the 72th and 84th months. However, we retained only 14 delivery dates in Table 7, in order to simplify the presentation.

5 See for example Singh (1997), Fryes (1997), or Hull (2003) for this kind of application in the case of interest rates.

6 Such an assumption is possible, even if these two studies do not rely on the same period of observation, if we consider, as Pérignon and Villa (2003) showed it, that there is a permanent nature of the factors affecting the dynamics of the term structure of interest rates.

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Appendix

Table 1A. **Principal components, standardized prices, 1989-2002.**

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6	Factor 7	Factor 8	Factor 9	Factor 10	Factor 11	Factor 12	Factor 13	Factor 14	Factor 15
1 month	0.2493	0.5121	-0.592	0.4931	0.2752	0.0691	0.0198	0.0289	-0.0079	0.0123	-0.0056	0.0061	-0.0019	-0.0001	-0.0002
2 months	0.2541	0.4062	-0.1723	-0.3489	-0.5813	-0.4607	-0.1029	-0.2349	0.053	-0.0149	0.0043	-0.0087	0.0035	-0.0022	0.0001
3 months	0.2571	0.3063	0.0817	-0.3906	-0.1302	0.4764	0.1258	0.6058	-0.2159	0.0796	-0.0277	0.0184	-0.0058	0.0005	0.0005
4 months	0.259	0.2166	0.2319	-0.2376	0.2797	0.3286	0.0621	-0.3922	0.4407	-0.3753	0.227	-0.1821	0.0811	0.0746	0.0062
5 months	0.2602	0.1392	0.2866	-0.1023	0.3084	-0.0205	-0.0468	-0.3109	-0.1134	0.3223	-0.4164	0.4363	-0.2967	-0.2387	-0.0221
6 months	0.2608	0.0699	0.2921	0.0399	0.2531	-0.2544	-0.1209	-0.0233	-0.3276	0.3354	0.0914	-0.287	0.5242	0.3317	0.0297
7 months	0.261	0.0082	0.2656	0.1551	0.1207	-0.3141	-0.1641	0.2531	-0.1417	-0.2018	0.3716	-0.2781	-0.5301	-0.2712	-0.0466
8 months	0.2609	-0.0476	0.2214	0.2275	-0.0363	-0.219	-0.1578	0.3533	0.2519	-0.443	-0.2134	0.4761	0.2514	0.1736	0.0678
9 months	0.2606	-0.0988	0.1593	0.266	-0.2201	-0.0757	0.8684	-0.1053	-0.0803	-0.0114	0.0131	0.0005	-0.0055	0,0000	0.0045
10 months	0.2602	-0.1471	0.084	0.2409	-0.2458	0.1505	-0.1698	0.091	0.4485	0.3047	-0.3169	-0.3979	0.1326	-0.3434	-0.1928
11 months	0.2596	-0.1918	-0.0009	0.1802	-0.2505	0.2484	-0.2016	-0.0992	0.1167	0.2945	0.2116	0.111	-0.3533	0.4941	0.4026
12 months	0.2589	-0.2328	-0.0912	0.0818	-0.1756	0.2442	-0.1875	-0.1986	-0.2711	-0.0428	0.3925	0.3314	0.1966	-0.1525	-0.5455
13 months	0.258	-0.2708	-0.1819	-0.0497	-0.0364	0.1365	-0.1145	-0.1653	-0.3717	-0.3172	-0.1952	-0.148	0.1686	-0.3383	0.5679
14 months	0.2571	-0.3067	-0.2695	-0.2017	0.1337	-0.0551	0.0215	-0.0052	-0.109	-0.2112	-0.4112	-0.2443	-0.2601	0.4417	-0.3963
15 months	0.256	-0.3402	-0.3548	-0.3565	0.3072	-0.2547	0.1686	0.2028	0.3285	0.269	0.275	0.1662	0.0955	-0.1698	0.1242