The Benefits of Hedge Funds in Asset-Liability Management

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Introduction

Recent difficulties have drawn attention to the risk management practices of institutional investors in general and defined benefit pension plans in particular. A perfect storm of adverse market conditions over the years 2000-2003 has devastated many corporate defined benefit pension plans. Negative equity market returns have eroded plan assets at the same time as declining interest rates have increased the marked-to-market value of benefit obligations and contributions. In extreme cases, corporate pension plans have been left with funding gaps as large as or larger than the market capitalisation of the plan sponsor. For example, in 2003, the companies included in the S&P 500 and FTSE 100 indices faced a cumulative deficit of $225 billion and £55 billion respectively (Credit Suisse First Boston 2003 and Standard Life Investments 2003), while the worldwide deficit reached an estimated $1,500 to $2,000 billion (Watson Wyatt 2003). Similar difficulties have been encountered by insurance companies, as declines in bond returns have encouraged them to seek performance potential in the equity asset class, at a time when the perceived risk was increasing significantly.

That institutional investors have been so dramatically affected by market downturns has led to major changes in institutional money management, including an increased focus on asset-liability management (ALM). In this context, institutional investors are desperately seeking new asset classes or investment styles that could be cast in a surplus optimisation context and would offer access to equity-like premiums without the associated downside risks.

Because of their focus on absolute performance and risk control, hedge funds are often recommended as a natural alternative to stocks and bonds. While long-only investment strategies can generate only a simple linear exposure to the return on underlying asset classes (they go up and down with the indices), the main benefit of hedge fund strategies is that they allow a convex non-linear exposure with respect to stock and bond returns in such a way that the downside risk is usually limited[1]. After all, hedge fund managers, who operate in the absence of regulatory constraints, can incorporate a variety of dynamic investment strategies and/or investments in derivatives likely to generate convex payoffs (Fung and Hsieh 1997). Although the benefits of including hedge funds in an investor's portfolio have been the object of much recent literature (*e.g.*, Agarwal and Naik 2004), these benefits have not been examined in an ALM framework. This paper is an attempt to undertake this examination.

From a conceptual standpoint, there are two possible approaches to the inclusion of hedge funds in ALM. A first approach to a formal model of the incorporation of hedge funds to ALM consists of treating hedge funds as a *supplement* to traditional asset classes, *i.e.*, as an additional asset class that can be added to stocks and bonds in a traditional ALM surplus optimisation exercise. In what follows (see section 4), we will argue that this approach, while seemingly straightforward, is too simplistic and involves a level of sample-dependence that is too high to be of any practical relevance, as is perhaps best evidenced by the unreasonably high (close to 100%) levels of optimal allocation to hedge funds it often leads to.

In this paper, we introduce a competing, more cautious, approach, which consists of treating hedge funds as a *complement*, as opposed to an *addition*, to traditional asset classes (see section 5). Overall, the results obtained with this more robust approach strongly suggest that, when mixed with bonds and stocks, suitably designed portfolios of hedge funds can allow for significant benefits in an ALM context, as can be measured in terms of reduction of the expected mismatch between assets and liabilities. This impact is more pronounced when the relevant optimisation objective includes a focus on extreme risks.

The rest of this paper is organised as follows. The next section presents an overview of asset-liability management techniques. In section 3, we present a formal surplus optimisation model. In section 4, we discuss conceptual and technical challenges to the introduction of hedge funds in the context of surplus optimisation. In section 5, we propose a novel approach that treats alternative investment

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strategies as a complement, as opposed to an addition, to traditional asset classes. In section 6, we present a series of formal numerical experiments and test for the impact of introducing hedge funds in terms of surplus optimisation benefits based on reasonable parameter values. A conclusion can be found in section 7, a list of references in section 8, while information on hedge fund indices is relegated to an appendix in section 9. Section 10 provides alternative investment figures for institutional investors.

I. An Overview of Asset-Liability Management Techniques

Asset-liability management (ALM) is the adaptation of the portfolio management process to the presence of various constraints relating to the commitments that figure in the liabilities of an institutional investor's balance sheet (commitments to paying pensions, insurance premiums, etc.). There are therefore as many types of liability constraints as there are types of institutional investors, and thus just as many approaches to asset-liability management.

ALM-type techniques fall into several categories. *Cash-flow matching* involves ensuring a perfect match between the cash flows from the portfolio of assets and the commitments in the liabilities. Suppose, for example, that a pension fund has promised to pay out a monthly pension. Leaving aside the complexity relating to the pensioner's uncertain life expectancy, the structure of the liabilities is defined simply as a series of cash outflows to be made, the real value of which is known today, but the nominal value of which is typically matched with an inflation index. It is possible in theory to construct a portfolio of assets whose future cash flows will be identical to this structure of commitments. To do so, assuming that such securities exist on the market, would involve purchasing inflation-linked zero-coupon bonds with a maturity corresponding to the dates on which the monthly pension benefits are paid out, with amounts that are proportional to the amount of real commitments.

This technique, which has the advantage of simplicity and in theory allows perfect risk management, nevertheless has a number of drawbacks. First, it will generally be impossible to find inflation-linked securities whose maturity corresponds exactly to the liability commitments. Moreover, most of those securities pay out coupons; there is then the problem of reinvesting the coupons. To the extent that perfect matching is impossible, there is a technique called *immunisation*, which allows the residual interest rate risk created by the imperfect match between the assets and liabilities to be managed optimally. This interest rate risk management technique can be extended beyond a simple duration-based approach to fairly general contexts, including hedging non-parallel shifts in the yield curve (see Martellini, Priaulet, and Priaulet 2003), or to simultaneous management of interest rate risk and inflation risk (Siegel and Waring 2004). It should be noted, however, that this technique is difficult to adapt to hedging non-linear risks related to the presence of options hidden in the liability structures (Le Vallois *et al.* 2003), and/or to hedging non-interest rate related risks in liability structures.

Another and probably greater disadvantage of cash-flow matching (or of the approximate matching version represented by immunisation) is that it represents a position that is extreme and not necessarily optimal for the investor in the risk/return space. In fact, cash-flow matching in asset-liability management is the equivalent of investing in the risk-free asset in asset management. It allows perfect management of the risks – a capital guarantee in the passive management framework – and a guarantee that the liability constraints are respected, but it makes piddling contributions to the assets.

To improve the profitability of the assets, it is necessary to make asset classes (stocks, government bonds, and corporate bonds) that are not perfectly correlated with the liabilities an integral part of the strategic allocation. It will then be a matter of finding the best possible trade-off between the risk (relative to the liability constraints) thereby taken on and the excess returns that the investor can hope to obtain through the exposure to rewarded risk factors. Different techniques are then used to optimise the surplus, *i.e.*, the value of assets in excess of that of liabilities, in a risk/return space, mostly relying on stochastic models that allow representation of the uncertainty relating to a set of risk factors that impact the liabilities. These can be financial risks (inflation, interest rate, stocks) or non-financial risks (demographic risks) are particularly useful. When necessary, agent behaviour models are then developed; it is then possible to show the impact of decisions linked to the exercise of certain implicit options. For example, an insured person can (usually in exchange for penalties) cancel his/her life insurance contract if the guaranteed contractual rate drops significantly below the interest rate prevailing at a date following the signature of the contract, a possibility that makes the amount of liability cash flows, and not just their current value, dependent on interest rate risk.

Different optimisation models are used by institutional investors for ALM (see Mulvey *et al.* 2005 for an example), and it is impossible to provide an exhaustive list here[2]. We now describe the specific model we use in this study.

II. A Formal Surplus Optimisation Model

A surplus optimisation model involves optimising the match between the asset and liability sides of financial structures in companies.

Instead of making assumptions on the detailed allocation to single assets or funds in investors' portfolios, one uses proxies for the different asset classes. In the context of this exercise, we consider three asset classes (in addition to hedge funds): stocks, nominal bonds, and inflation-indexed bonds (TIPS). The portfolio return R_{PF} is then given as:

$$R_{PF,t} = \sum_{i=1}^{n} \omega_i R_{i,t} \qquad t = 1 \ldots T$$

where i represents the proxy for the asset class i and ω_i its weight in the portfolio.

The purpose of surplus optimisation is to find the allocation that minimises at horizon T (here taken to be ten years) the relative expected shortfall SF beyond a certain target α, which is defined as follows:

$$SF(\alpha) = -E\left(\frac{R_{PF,T} - L_T}{L_T}\middle|\frac{R_{PF,T} - L_T}{L_T} < \alpha\right).$$

In the remaining sections, the optimal allocation will be obtained by resolving the following objective function:

$$\omega^* = \arg\min_{\omega} SF(0)$$

To optimise expected values of the portfolio distribution, we need to generate stochastic scenarios for both the asset and liability sides from an *ex-ante* basis. On the asset side, Monte-Carlo analysis is used to generate 10,000 random paths for each asset class using geometric Brownian motions and we generate scenarios.

$$S(t) = S(0)\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B(t)\right)$$

where B(t) is a Brownian motion with $\Delta B(t) \sim N(0,\Delta t)$ so that:

$$S(t+s) = S(t)\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sqrt{s}\,N\right)$$

with $N \sim N(0,\sigma^2)$.

To take the correlation of the underlying asset classes into account we will introduce the 3-dimensional geometric Brownian motion:

$$S_a(t+s) = S_a(t)\exp\left(\left(\mu_a - \frac{1}{2}\sigma_a^2\right)t + \sqrt{s}\,N_a\right)$$
$$S_b(t+s) = S_b(t)\exp\left(\left(\mu_b - \frac{1}{2}\sigma_b^2\right)t + \sqrt{s}\,N_b\right)$$
$$S_c(t+s) = S_c(t)\exp\left(\left(\mu_c - \frac{1}{2}\sigma_c^2\right)t + \sqrt{s}\,N_c\right)$$

with the 3-dimensional Gaussian:

$$\begin{pmatrix} N_a \\ N_b \\ N_c \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_a^2 & \sigma_a\sigma_b\rho_{ab} & \sigma_a\sigma_c\rho_{ac} \\ \sigma_a\sigma_b\rho_{ab} & \sigma_b^2 & \sigma_b\sigma_c\rho_{bc} \\ \sigma_a\sigma_c\rho_{ac} & \sigma_b\sigma_c\rho_{bc} & \sigma_c^2 \end{pmatrix}\right)$$

Long-term estimates are used to calibrate the model (see exhibit 1)[3]. For mean return and volatility on stocks, bonds, and TIPS, we have used Dimson, Marsh, and Staunton's (2002) 1900-2000 estimates; for volatility on TIPS, and for the correlation matrix, we have used Kothari and Shanken's (2004) 1953-2000 estimates[4].

Exhibit 1. Long-term parameter estimates

Correlation	Stocks	Bonds	TIPS
Stocks	1		
Bonds	0.24	1	
TIPS	– 0.05	0.52	1
Mean	**10.4%**	**5.8%**	**4.3%**
Volatility	**16.5%**	**8.5%**	**6.58%**

As explained in section 4, we do not attempt to estimate long-term parameter values for hedge funds, as we believe that doing so would be of little relevance. We will instead model the introduction of hedge funds through their impact on these long-term parameter values for stocks and bonds; in particular, we will estimate the decrease in stock and bond volatility that can be achieved by including suitably selected hedge fund strategies in an institutional investor's stock or bond allocation (see section 5).

On the liability side, when we examine pension funds (section 6.1), we will assume that returns on liabilities are equal to those on inflation-indexed bonds plus 300 basis points, making it possible to take into account the main risk factors, *i.e.*, inflation and interest rates. As a result, liabilities are perfectly correlated with the return on TIPS in our model. In practice, TIPS are certainly more closely correlated with liabilities than any other asset class, even though the correlation is imperfect. After all, there are a number of extraneous sources of risk (*e.g.*, actuarial risk) in addition to inflation and interest rates. In life-insurance contract management, examined in section 6.3, we will propose a stylised formalisation of liability flows, taking into account key elements such as profit-sharing and the existence of surrender clauses.

Generally speaking, the relative expected shortfall is given as:

$$SF(0) = -\frac{1}{n}\sum_{s=1}^{10000}\left(\frac{R^s_{PF,T} - L^s_T}{L^s_T}\cdot 1\{R^s_{PF,T} - L^s_T < 0\}\right)$$

where the exponent *s* denotes the scenario and *n* is the number of scenarios yielding deficits after 10 years:

$$n = \sum_{s=1}^{10000}\left(1\{R^s_{PF,T} - L^s_T < 0\}\right)$$

III. Allocation to Hedge Funds in the Context of Surplus Optimisation: the Naïve Approach

There are two possible approaches that lead to the inclusion of hedge funds in ALM. One involves treating hedge funds as a *supplement* to traditional asset classes, while the other treats them as a *complement*.

In this section, we will argue that, while the first approach is a straightforward attempt to formalise the benefits of hedge funds in ALM, it must nonetheless overcome a number of conceptual and technical obstacles. First, it is based on the assumption that hedge funds can be treated as a coherent asset class; conceptually, this assumption is unsatisfactory, as hedge funds include a set of very diverse investment strategies. Secondly, from a technical standpoint, it must be recognised that there are not yet any truly satisfactory models of the dynamics of hedge fund returns that can be used in an *ex-ante* Monte Carlo simulation. In addition, even if there were such models the lack of a long history of hedge fund returns and various concerns about the quality of hedge fund return data would make the estimation of reliable parameter values a very challenging task.

III.1. CHALLENGES IN MODELLING HEDGE FUND RETURNS

That hedge funds have started to gain widespread acceptance while remaining somewhat mysterious investment vehicles has increased the need for better measurement and benchmarking of their performance. Although attempts to understand the risk exposures of hedge funds have become common in academic research, a satisfactory description of the dynamics of hedge fund returns has yet to be developed. The nature of risks associated with hedge fund strategies is complex. In particular, since hedge fund returns exhibit non-linear option-like exposures to traditional asset classes (Fung and Hsieh 1997, 2000), standard asset pricing models offer limited help in evaluating the performance of hedge funds. The importance of taking into account such option-like features has been underlined by recent research. Fung and Hsieh (2002) and Mitchell and Pulvino (2001) stress taking into account option-like features while analysing the performance of "trend-following" and "risk-arbitrage" strategies. More recently, Agarwal and Naik (2003) build on these insights and extend our understanding of hedge fund risks to a wide range of equity-oriented hedge fund strategies. They characterise the risk exposures of hedge funds using buy-and-hold and option-based strategies and show that a large number of equity-oriented hedge fund strategies exhibit payoffs resembling a short position in a put option on the market index (see also Schneeweis and Spurgin 2000 and Fung and Hsieh 2001 for related papers).

There are actually two possible ways to try to adapt standard asset pricing models to analyse returns on portfolios that exhibit non-linear dependency with respect to standard asset classes. The first approach involves a non-linear APT model (see Bansal and Viswanathan 1993 or Bansal, Hsieh, and Viswanathan 1993). The other method, used by Glosten and Jagannathan (1994), as well as in the papers mentioned in the previous paragraph, is to include new regressors with non-linear exposure to standard asset classes, *e.g.*, returns on option positions, to proxy for dynamic trading strategies in a linear regression and thus better understand return sources.

Return-based style (RBS) factors and asset-based-style (ABS) factors can both be used to find the significant factors that account for returns. RBS factors refer to the notion of an implicit factor model. They are obtained through principal component analysis. The aim is to account for the return series of observed variables through a smaller group of non-observed implicit variables. The implicit factors are extracted from the time-series of returns, with each implicit factor defined as a linear combination of the primary variables. The advantage of this approach is that it allows the user to avoid the risk of under-specifying the model (omitting true factors) or over-specifying it (including spurious factors). The drawback relates to the economic significance of the implicit variables obtained. ABS factors refer to the notion of an explicit factor model. In this approach the specification of the model plays an important role. A discretionary choice of observable market risk factors is made, and the risks of misspecification are non-negligible. In addition, it is easier to interpret the factors in the model[5].

There are a number of papers that in this strand of the literature, including Mitchell and Pulvino (2001) for the replication of risk arbitrage returns, Fung and Hsieh (2001) for the trend-following strategy, Fung and Hsieh (2002) for the fixed-income arbitrage strategy, Fung and Hsieh (2003) for the modelling of long/short equity, Fung and Hsieh (2004) for a diversified hedge fund portfolio, Agarwal, Fung, Loon, and Naik (2004) for the modelling of convertible arbitrage returns, and Karavas, Kazemi, and Schneeweis (2004) for the modelling of European-based hedge fund managers.

While the aforementioned papers make progress in the direction of modelling hedge fund returns, investors do not yet have at their disposal a set of models that can be used to generate stochastic scenarios for various hedge fund strategies. A review of studies of hedge fund performance modelling shows that the performance and quality of replication is relatively low, especially from an out-of-sample perspective (see Karavas, Kazemi, and Schneeweis 2004). This stands in sharp contrast to the situation for traditional asset classes, where satisfactory, albeit imperfect, models are available (in particular stochastic volatility models for stock prices and multi-factor models of the term structure of interest rates for bond prices). This technical obstacle can be partially overcome with non-parametric bootstrapping techniques that make it possible to generate a multitude of scenarios based on random sampling of some history of hedge fund returns. The latter solution is not fully satisfactory, however, as it is highly sample-dependent. As it happens, the problem of sample dependency of parameter estimates is what we turn to next.

III.2. CHALLENGES IN ESTIMATING PARAMETERS FOR HEDGE FUND RETURN DISTRIBUTIONS

Since the hedge fund industry is relatively young, reliable data on hedge fund index returns have been available only since the early 1990s. Hedge fund indices are built from databases of individual funds and then inherit their shortcomings in terms of scope and quality of data. Hedge fund indices suffer from several biases. The first, a result of voluntary participation in databases, is the "self-reporting bias". Since the funds that have refused to report to a database are, by definition, unobservable, it is impossible to quantify the impact of this bias or even to know whether it has a positive or negative impact on performance. Second, the lack of transparency also calls into question the reliability of data and exposes investors to a risk of change in the manager's style, known as "style drift" (Lhabitant 2001). This impact of this bias is likewise difficult to evaluate. In addition, hedge funds may decide to register in order to communicate, as they do not have the right to advertise, but with no intention of disclosing information on a regular basis, a practice that tends to smooth the returns and underestimate volatility. "Survivorship bias" is caused by the possible removal from the database of the data on funds that have defaulted or stopped reporting. Not all databases are affected in the same way by this bias. For example, the TASS database has a higher survivorship bias than the HFR database because it has a higher attrition rate. Finally, the funds have selection criteria that can be very diverse, and the data provided will not be representative of the same management universe. This is referred to as "selection bias". Of the 1,162 HFR funds and the 1,627 TASS funds, only 465 are common to both databases (Liang 2001).

III.3. A SIMPLE ILLUSTRATION

The following simple *ex-post* experiment illustrates the remarkable lack of robustness induced by a naïve use of hedge fund returns in the context of relative risk optimisation.

Assuming for simplicity's sake, as explained in section 3, that the return on liabilities is equal to the return on inflation-indexed bonds to which we add 300 basis points, we optimise the information ratio of the asset portfolio, composed of stocks, bonds, and hedge funds, with respect to the liabilities, defined as the excess return divided by tracking error. Arguably, this simple experiment does not entirely describe the flavour of a full-fledged asset-liability model. However, it does illustrate the lack of robustness implied by a straightforward use of hedge fund return data in portfolio optimisation.

Exhibit 2 shows the result of the maximisation of the information ratio based on a rolling-window on the out-of-sample period ranging from 04/2000 to 03/2005

As exhibit 2 shows, the portfolio is almost totally invested in stocks until mid-2001, *i.e.*, until stock markets begin to fall. Then, until early 2003 and again from the last quarter of 2003 and to the end of the study period, it becomes totally invested in hedge funds. During the remainder of 2003, the portfolio is totally invested in bonds. These extreme allocations lead to total rebalancing of the portfolio every time market conditions change.

The results, which suggest that the different asset classes (stocks, bonds, and hedge funds) should represent either 0% or 100% of an optimal allocation, are typical of unintuitive, highly-concentrated, input-sensitive portfolios that are obtained from naïve optimisation procedures. It is, of course, hardly reasonable to invest the entire portfolio in hedge funds, considered the riskiest asset class. Such a configuration will certainly not be viable out of sample. The results suggest that a straightforward use of hedge fund returns in an optimisation procedure is not a satisfactory response to the questions relating to the role of hedge funds in ALM. We now turn to a pragmatic alternative approach to the question.

■ IV. Allocation to Hedge Funds in the Context of Surplus Optimisation: a More Robust Approach

For all the conceptual and technical reasons outlined in the previous section, we advocate a competing approach in this paper that does not treat hedge funds as an *addition* but rather as a *complement* to traditional asset classes (stocks and bonds). This alleviates the concern over *ex-ante* modelling of hedge fund returns: hedge funds only enter the surplus optimisation exercise through the impact they have on risk parameter estimates for stocks and bonds, through a reduction in the risk measures for traditional asset classes. On the other hand, we choose not to add hedge funds to the TIPS component of the investor's portfolio. After all, including hedge funds would have a negative impact

Exhibit 2.

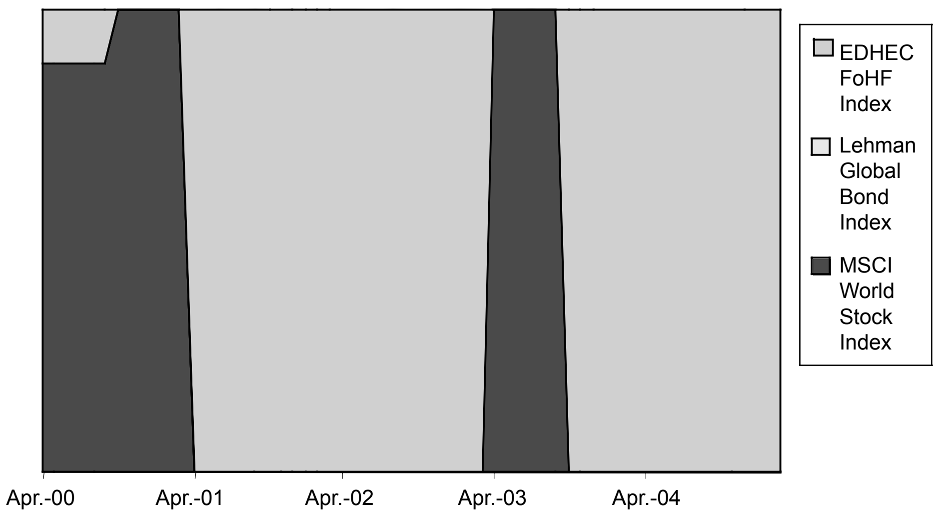


Exhibit 2. This exhibit shows the optimal allocation to stocks, bonds, and hedge funds, for an investor seeking to maximise his historical information ratio with regards to liabilities equal to the return on inflation-indexed bonds to which we add 300 basis points. The EDHEC Fund of Hedge Funds Index, the Lehman Brothers Bond Index, and the MSCI World Index are used as proxies for hedge fund returns, bond returns, and stock returns.

on the correlation between the TIPS portfolio and the liability returns, when the reason for the existence of the TIPS portfolio is precisely to show very high correlation with the investor's liabilities (perfect correlation in our simplified model of liabilities) for which it serves as a natural hedge.

IV.1. RISK REDUCTION BENEFITS OF HEDGE FUNDS

Previous research has shown that some (but not all) hedge fund strategies mix well with either stocks or bonds in terms of risk reduction benefits, where risk is measured not only in terms of portfolio volatility but also in terms of impact on higher moments of portfolio distribution.

We reproduce below results obtained by Amenc *et al.* (2005), who suggest constructing multi-strategy hedge fund benchmarks that would exhibit a persistent and robust factor exposure and meet the needs of different classes of investors. In particular, we aim to design two separate hedge fund portfolios: an equity diversifier hedge fund benchmark, meant to diversify an equity portfolio, and a bond diversifier hedge fund benchmark, meant to diversify a bond portfolio. The design of these benchmarks again involves separate selection and allocation phases.

IV.1.1. Selection Phase

In the selection phase, we look at the diversification properties of hedge fund strategies with respect to portfolios of stocks or bonds. Because of evidence that hedge fund returns are not normally distributed, we look beyond the first and second order moments of hedge fund return distributions when searching for strategies with good diversification properties.

Since it is widely accepted that investors have a non-trivial preference for higher moments of the return distribution, it is crucial to assess how an asset contributes to these different moments. The beta for all four moments can be calculated. The second moment beta is the contribution of an asset to the second moment (volatility) of the portfolio when a small fraction of this asset is added. This corresponds to the standard CAPM beta commonly used in investment analysis. The third moment beta and the fourth moment beta contribute to the portfolio's third and fourth moments. The table below shows values for the beta for the most important hedge fund strategies, when these strategies are added to a portfolio of equities or bonds. In general, the lower the beta for a given strategy, the higher the diversification benefits when this strategy is added to a portfolio of conventional assets. The addition of a small fraction of a new asset (*e.g.*, a hedge fund) to a portfolio leads to a decrease in the portfolio's second moment only if the second moment beta is less than 1, to an increase in its third moment only if the third moment beta is less than 1, and to a decrease in its fourth moment only if the fourth moment beta is less than 1 (see Martellini and Ziemann 2005 for greater detail)[6].

For these results, we have used EDHEC Alternative Indices as proxies for the return on hedge fund strategies. The appendix in section 9 is devoted to a brief presentation of these indices.

Considering these results, as in Amenc *et al.* (2005), we select a sub-set of three strategies to construct the diversification benchmark aimed at diversifying equity-oriented portfolios and a sub-set of four strategies to construct the diversification benchmark aimed at diversifying bond-oriented portfolios. The table below shows the result of the selection process. The strategies that are selected are marked "Yes" in the column corresponding to the respective diversifier benchmark.

Exhibit 3: Higher moment beta of returns of several hedge fund strategies (as represented by EDHEC Alternative Indices) with stock and bond returns (as represented by the MSCI World indices for sovereign bonds and equity). Based on monthly returns for the period 01/1997 to 12/2006

	Convertible Arbitrage	CTA Global	Event Driven	Long/ Short Equity	Equity Market Neutral
2nd Moment Beta with Equity	0.06	– 0.11	0.27	0.38	0.06
2nd Moment Beta with Bonds	– 0.06	1.51	– 0.34	– 0.37	0.05
3rd Moment Beta with Equity	0.06	– 0.32	0.43	0.37	0.06
3rd Moment Beta with Bonds	1.23	– 1.01	1.39	0.77	0.65
4th Moment Beta with Equity	0.10	– 0.26	0.36	0.38	0.07
4th Moment Beta with Bonds	– 0.12	1.27	– 0.36	– 0.08	0.08

Exhibit 4: Strategies Entering the Equity and Bond Diversifiers

Investable Index	Equity Diversifier	Bond Diversifier
Convertible Arbitrage	Yes	Yes
CTA Global	Yes	No
Equity Market Neutral	Yes	Yes
Event Driven	No	Yes
Long/Short Equity	No	Yes

IV.1.2. Optimisation Phase

The next step is to find the optimal allocation of the selected strategy indices. As in Amenc *et al.* (2005), our methodology is based on the following two key principles:

- *Principle 1: Because expected returns are notoriously hard to estimate with any degree of accuracy, the focus is on minimising the risk of an investor's overall portfolio (stock or bond).*
- *Principle 2: Because hedge funds are not normally distributed, the measure of risk used should be more general than volatility.*

In what follows, we carry out a risk minimisation calculation, in which we use the VaR at a threshold of 95%, integrating the Cornish-Fisher correction that allows us to take investors' aversion to extreme risks into account. Furthermore, we constrain the weight of the hedge fund portfolio to take on different values (5%, 15%, 25%, 35%) of the investor's global allocation, with remaining wealth fully invested in either bonds or stocks.

The tables below (exhibits 5 and 6) from Amenc *et al.* (2005) show the diversification benefits obtained by adding the diversification benchmarks to a stock or bond portfolio. The first column shows the performance statistics for the stock and bond indices. The columns to the right show the same statistics when adding the diversifier at different weights.

From the numbers in these tables, it becomes clear that even with a small percentage allocated to hedge funds, an investor achieves economically significant diversification benefits. For an equity investor, including hedge funds at a weight of 15% in the suggested way, monthly Value-at-Risk and portfolio volatility are reduced by a considerable 15%, volatility is reduced from 15% to 12.5%, and the Cornish-Fisher VaR from 7.6% to 6.2%, while the mean return increases by more than 30% (from 2% to 2.7%). For a bond investor, the mean return improves slightly, while the risk declines by more than 12% in terms of VaR and by more than 15% in terms of volatility.

In the long-term ALM allocation exercise in the next section, we model the reduction of the long-term volatility parameter for stocks and bonds achieved by using hedge funds in surplus optimisation. We assume that the reduction in volatility obtained on the sample 10/2001 to 09/2004 is a robust indication of what can be obtained over the longer term[7]. Based on estimates from exhibits 5 and 6 for estimates of decreases in volatility induced by hedge funds, and on long-term estimates of stock and bond volatility in the absence of hedge funds borrowed from exhibit 1, we obtain the corresponding parameter (σ) estimates as a function of the portion of hedge funds added to the traditional asset classes (exhibit 7).

Numbers in exhibits 5 and 6 show that in the sample from 10/2001 to 09/2004 adding 5% to 35% of a suitably designed portfolio of hedge funds (or *equity diversifier*) to stocks reduces the volatility of that asset class to 37.33%, a reduction of 5.33%. We apply these reduction coefficients to the long-term estimate (16.50% volatility estimate for stocks; see exhibit 1) to obtain a reduction in these values from 15.62% (for a 5% addition of hedge funds) to 10.34% (for a 35% addition of hedge funds). The respective risk reduction for bonds comes to 6.06% to 27.27% when a suitably designed

Exhibit 5. Portfolio performance when an equity diversifier is added to the MSCI World Equity Index

Allocation to Hedge Funds	0%	5%	15%	25%	35%
Annualised Mean Return	2.0%	2.2%	2.7%	3.2%	3.7%
Annualised Std Deviation	15.0%	14.2%	12.5%	10.9%	9.4%
VaR (95%)	7.6%	7.1%	6.2%	5.3%	4.4%
Sharpe Ratio (risk-free rate = 2%)	– 0.03	0.015	0.057	0.111	0.184
Skewness	– 0.56	– 0.54	– 0.49	– 0.41	– 0.30
Kurtosis	3.27	3.24	3.17	3.08	2.95

Summary statistics for a portfolio composed of the MSCI World Equity Index and an optimal diversifier. Allocation to hedge funds (= optimal diversifier) ranges from 5 to 35%. The 0% case is shown for comparison purposes. The diversifier is constructed by minimising the 95% Cornish-Fisher VaR of the overall portfolio. It is composed of EDHEC indices for Convertible Arbitrage, CTA Global and Equity Market Neutral. Weights of a single index are constrained to a maximum of 40% in the optimal diversifier. The computations come from Amenc et al. (2005) and are based on monthly return data from 10/2001 to 09/2004.

Exhibit 6. Portfolio performance when bond diversifier is added to the Lehman Composite Global Treasury Index

Allocation to Hedge Funds	0%	5%	15%	25%	35%
Annualised Mean Return	– 0.3%	0.1%	0.9%	1.8%	2.6%
Annualised Std Deviation	3.3%	3.1%	2.8%	2.6%	2.4%
VaR (95%)	1.7%	1.5%	1.3%	1.1%	0.9%
Sharpe Ratio (risk-free rate = 2%)	– 0.71	– 0.61	– 0.38	– 0.09	0.25
Skewness	– 0.26	– 0.23	– 0.13	0.05	0.25
Kurtosis	2.40	2.51	2.82	3.24	3.65

Allocation to hedge funds (= optimal diversifier) ranges from 5 to 45%. The 0% case is shown for comparison purposes. The diversifier is constructed by minimising the 95% Cornish Fisher VaR of the overall portfolio. It is composed of EDHEC indexes for Convertible Arbitrage, Event Driven, Long/Short Equity and Equity Market Neutral. Weights of a single index are constrained to a maximum of 30% in the optimal diversifier. The computations come from Amenc et al. (2005) and are based on monthly return data from 10/2001 to 09/2004.

portfolio (or *bond diversifier*) is used. We again apply these reduction coefficients to the long-term estimate (8.50% volatility estimate for bonds; see exhibit 1) to obtain a reduction in these values from 7.98% (for a 5% addition of hedge funds) to 6.18% (for a 35% addition of hedge funds). The numbers that appear in exhibit 7 will be used in the surplus optimisation exercise (section 6).

It should be emphasised that we have chosen *not* to consider the impact on expected returns in the context of this surplus optimisation exercise. While the addition of hedge funds to a stock or bond portfolio is likely to have a non-trivial impact on performance (a positive impact in this sample), intuition suggests that such return enhancement benefits may not necessarily be very robust and should not be incorporated in a long-term allocation exercise. That intuition has been formalised in Martellini, Vaissié, and Ziemann (2005), who have shown that hedge fund ability to diversify traditional asset portfolios in terms of reduction in both variance and kurtosis is rather robust through time, while benefits in terms of increases in expected returns and decrease in skewness are less stable[8]. This legitimises the focus on the impact of hedge funds on risk reduction, as opposed to return enhancement[9].

V. Numerical Results

It is our aim to take three examples to illustrate numerically the benefit of including hedge funds in ALM. We will first study a generic exercise concerning an individual retirement liability like the PERP (Plan d'Epargne Retraite Populaire), characterised by a relatively long investment horizon and high exposure to inflation risk. Second, we will examine an ALM problem with a shorter horizon (*e.g.*, four years), a horizon over which hopes that long-term phenomena such as the reversion to the mean of market returns can lower the risk of extreme loss are unreasonable[10]. Finally, we will look into life insurance (euro contracts) to show how hedge funds allow asymmetric management of interest rate risk.

Exhibit 7. Evolution of volatility of stocks and bonds as a function of the proportion allocated to hedge funds in the stock and bond portfolios. The volatility estimates for stocks and bonds in the base case without hedge funds (0% HF) are taken from exhibit 1

	0% HF	5% HF	15% HF	25% HF	35% HF
Stocks	16.50%	15.62%	13.75%	11.99%	10.34%
Bonds	8.50%	7.98%	7.21%	6.70%	6.18%

V.1. INDIVIDUAL RETIREMENT LIABILITIES

Here, the objective is to perform an ALM exercise for long-term retirement liabilities that bear significant inflation risk. The investor's liabilities are a sum of nominal cash flows of fixed amounts to be paid each year for ten years. Below, we quantify the diversification and thus the reduction of risk (including expected shortfall risk) achieved by an increase in the share allocated to hedge funds in the equity and bond components of a portfolio.

For each of these parameter values, we create 10,000 scenarios and run the optimisation problem described in section 3, in which mean and correlation parameter values for stocks, bonds, and TIPS, as well as the volatility estimate for TIPS, are borrowed from exhibit 1, and volatility estimates for stocks and bonds vary as a function of the percentage allocated to hedge funds, as expressed in exhibit 7. For the optimal portfolio allocation as well

Exhibit 8. Evolution of optimal asset allocation, expected relative shortfall and probability of a shortfall greater than 10% as a function of the proportion allocated to hedge funds in the stock and (nominal) bond portfolios

HF Allocation within Stocks and Bonds	Stocks	Bonds	TIPS	Actual HF Allocation	Exp. Relat. Shortfall	Benefit of HFs	Prob (SF > 10%)	Benefit of HFs
0% HF	16.21%	26.40%	57.39%	0.00%	15.44%	**–**	53.75%	**–**
5% HF	16.65%	27.85%	55.49%	2.23%	15.30%	**0.90%**	53.08%	**1.25%**
15% HF	21.35%	20.55%	58.10%	6.29%	14.50%	**6.11%**	48.82%	**9.17%**
25% HF	25.18%	19.99%	54.83%	11.29%	13.94%	**9.71%**	44.24%	**17.69%**
35% HF	31.60%	13.55%	54.85%	15.80%	12.95%	**16.09%**	37.59%	**30.07%**

as the benefits in terms of relative expected shortfall and probability of extreme losses greater than 10%, we obtain the results in exhibit 8.

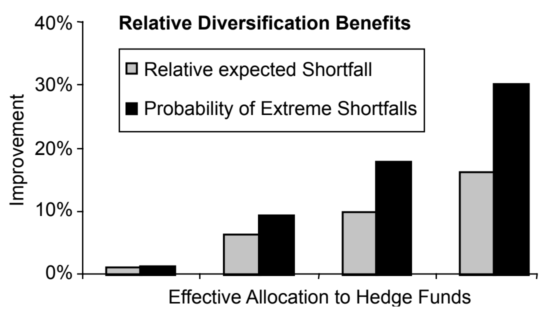
From these numbers, we find that the introduction of hedge funds allows significant improvement in risk management in an ALM context. For example, making hedge funds 25% of the stock and bond allocation portions of the investor's portfolio leads to a decrease of 9.71% in the expected shortfall. The impact on extreme risks is even more spectacular (the probability of a shortfall greater than 10%, *i.e.*, a deficit worse than 90%, falls by 17.69%).

A 25% allocation to hedge funds might be perceived as unusually high. It should be noted, however, that the implicit introduction of hedge funds has been done only for stocks and bonds, not for TIPS. Consequently, the actual amount invested in hedge funds is lower than mentioned in the tables above (5%-35%). The fifth column of the table in exhibit 8 shows the actual allocation to hedge funds as a function of the portfolios obtained. For example, having hedge funds account for 25% of the stock and bond portions of the portfolios leads to a total portfolio allocation to hedge funds of only 11.29%.

Exhibit 9 summarises the relative benefits in terms of expected relative shortfall and probability of extreme losses greater than 10%.

Overall, these results strongly suggest that, when added to bonds and stocks, suitably designed portfolios of hedge funds have significant benefits on ALM, as can be measured by the reduction of the expected mismatch between assets and liabilities. This impact on extreme risks is more spectacular still. Our most important finding is perhaps that even limited levels of investment in hedge funds allow significant decreases in extreme risks. In fact, we show that the probability of extreme deficits (the value of the assets falling below 90% of the value of liabilities) can be reduced by as much as 30% by allocating no more than 15% to hedge funds.

Exhibit 9. Improvement of expected relative shortfall and probability of a shortfall greater than 10% as a function of the proportions allocated to hedge funds



V.2. SHORT-TERM LIABILITIES

Here, we look at liabilities of relatively short duration (about four years, for example). Intuitively, it seems that the stakes for risk control are greater over the short term, in particular when equity risk investing is involved, as it is unreasonable to expect that reversion to the mean will mitigate the effect of significant losses on equity markets.

To evaluate the protective effect provided by including hedge funds, we first do the same optimisation as before but with four-year rather than ten-year liabilities. Table 10 (below) sums up the optimal allocation we obtained.

In the short term, and with significantly higher extreme risk than in traditional stochastic modelling, it is clear that for asset-liability management volatility alone is an insufficient measure of the risk of the equity component of a portfolio.

Table 10. Change in optimal asset allocation, expected relative shortfall and probability of a shortfall greater than 10% as a function of the proportion allocated to hedge funds in the stock and (nominal) bond portfolios.

HF Allocation in Stocks and Bonds	Stocks	Bonds	TIPS	Actual HF Allocation	Exp. Relat. Shortfall	Benefit of HFs	Prob(SF > 10%)	Benefit of HFs
0%	11.20%	10.19%	78.60%	0.00%	6.76%	**–**	16.00%	**–**
5%	10.97%	11.17%	77.86%	1.11%	6.63%	**1.88%**	15.39%	**3.81%**
15%	12.07%	11.33%	76.60%	3.51%	6.54%	**3.24%**	14.03%	**12.31%**
25%	15.74%	9.56%	74.71%	6.32%	6.36%	**5.97%**	13.63%	**14.81%**
35%	18.03%	4.54%	77.43%	7.90%	6.14%	**9.17%**	11.53%	**27.94%**

Second, we use a stress test, including a period of considerable stock market losses (April 2000 to March 2003), to analyse the capacity of hedge funds to reduce extreme risks with regards to equities, and thus the lowered probability of extreme shortfall. For this purpose, we consider an investor with a funding ratio of 100% in April 2000 and an allocation policy of assigning variable proportions to hedge funds, weighted as shown in table 10. Table 11 shows the value of expected relative shortfall at the end of the period of falling stock markets (March 2003) for different levels of investment in hedge funds.

It thus appears that an effective allocation of 7.90% in hedge funds, corresponding to a 35% allocation within the equity and nominal bond class, makes it possible to reduce in-sample the observed relative shortfall of about 15% during a period particularly unfavourable for stock markets. These results confirm the benefits of hedge funds when it comes to ALM risk management, benefits that were already observed in the setting of Monte Carlo analyses described in section 6.1.

V.3. LIFE-INSURANCE LIABILITIES

We look now at life-insurance liabilities, and we show how hedge funds can allow asymmetric management of interest rate risk. Here, the aim is to illustrate the ability of hedge funds to improve extreme risks of fixed-income products, used as an imperfect hedge for the liabilities of euro contracts, specifically when taking into account the possible modification of liabilities in the event of interest rate hikes[11].

We look at simplified life insurance liabilities, with a contract concerning 1,000 initial subscribers. For a maximum stylisation as well as an analytic compre-

Table 11. Value of expected relative shortfall at the end of the period of falling stock markets (March 2003), with an initial shortfall null in April 2000, as a function of the proportion allocated to hedge funds in the stock and (nominal) bond portfolios

Stocks	Bonds	TIPS	Exp. Relat. Shortfall with HFs	Exp. Relat. Shortfall without HFs	Benefit of HFs
11.20%	10.19%	78.60%	20.97%	20.97%	**0.00%**
10.97%	11.17%	77.86%	21.00%	20.72%	**1.35%**
12.07%	11.33%	76.60%	21.78%	20.86%	**4.42%**
15.74%	9.56%	74.71%	23.95%	21.96%	**9.08%**
18.03%	4.54%	77.43%	24.56%	21.39%	**14.87%**

We used the MSCI World index as a proxy for equity, the Lehman Global Treasury index as a proxy for nominal bonds, the Merrill Lynch US Treasury Inflation-Linked Bonds index as a proxy for real equity, and the EDHEC Fund of Funds index as a proxy for alternative assets.

hension of the various components of the ALM risk of a life-insurance company, we will do this study on a product with a fidelity bonus. This product will consist of a single premium of €100 paid down at the initial date, credited with a technical rate of 2% a year; profit-sharing is paid at term (assumed to be equal to eight years) if the client is faithful. Profit-sharing involves a payment of 90% of the portfolio performance, net of technical interest. For simplicity's sake, we do not model outstanding commissions; the result of the insurance company is observed at $t = 8$, as a function of the portfolio performance.

The choice of a product for which all profit shares are paid at the term of the contract allows a basic understanding of life-insurance mechanisms. In particular, it makes it possible to avoid having to define profit-sharing rules for the life of the contract, a situation that means defining the appreciation or depreciation realisation programme as well as the use of smoothing reserves, such as the surplus provisions, the modelling of which varies from company to company. Since we are concerned only with the final value of the contract, this study will also not consider the reserves for overall depreciation of assets[12].

In addition to his share in profits, the customer profits from the usual guarantees on his life insurance contract, in particular the possibility of surrendering his contract at any time, at the value given by the mathematical provisions (credited with the technical interest rate at any time). Here, it appears that the function of surrender is defined as a put on the value of the portfolio, whose strike price is equal to the mathematical provisions.

To take into account in the aggregate effects of tax and wealth issues, which have an impact on the decision to exercise the put and make it less of a rational exercise based on purely financial considerations, we study the more realistic case in which the behaviour of the policy-holders is modelled as a function of surrender. We thus assume that surrender rates rise as the fund depreciates.

More formally, the proportion of subscribers on date t is noted by p_t. The modelling of this proportion as a function of the difference between $PM(t)$ and $A(t)$ is shown below:

$$p_t = 0.8 \times \frac{\exp\left(-20 \times \left(\frac{A_t}{PM_t} - 0.8\right)\right)}{1 + \exp\left(-20 \times \left(\frac{A_t}{PM_t} - 0.8\right)\right)}$$

where A_t is the market value of assets at t, and PM_t the mathematical reserves at t given by $PM_t = PM_0 \times (1+r)^t$ for $t < 8$, and for $T = 8$ by the following relation integrating the function of profit-sharing: $PM_T = PM_0 \times (1+r)^T + \delta \times (A_T - PM_0 \times (1+r)^T)^+$. Here, r is the technical rate (assumed to be 2%) and δ is the rate of participation in the financial results (assumed to be 90%).

Exhibit 12 (below) makes it possible to visualise surrender rates at different sizes of the gap between the value of the portfolio and that of mathematical reserves.

We will then assume, as usual, that surrenders take place in proportion to both the number of customers and the value of mathematical provisions. These surrenders involve sales of assets, with the assumption that they maintain constant the fractions allocated to each asset clases.

For liability flows, F_t, we have:

$$F_t = PM_0 \times p_t \times (1+r)^t \quad for \quad t < 8$$

$$F_T = PM_0 \times \left(1 - \sum_{t=1}^{T-1} p_t\right) \times (1+r)^T + \delta\left(A_T - PM_0 \times \left(1 - \sum_{t=1}^{T-1} p_t\right) \times (1+r)^T\right)^+$$

The technical difficulty of the optimisation exercise is that the surrender function depends here on the value of the funds, and thus on the allocation strategy. Thus, the objective (expected shortfall, or expected relative shortfall) becomes a highly non-convex function of the variables of control (asset allocation sgtrategy). As the objective

Exhibit 12: Surrender rates as a function of the difference between the value of the portfolio and the value of the mathematical reserves

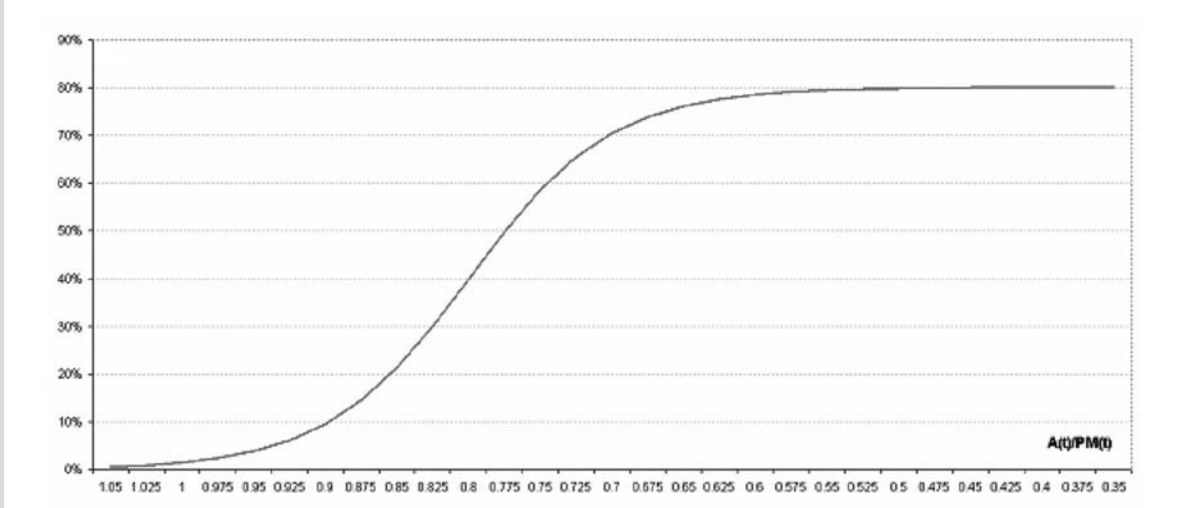


Exhibit 13. Presentation of the allocation strategies selected

Allocations	case 1	case 2	case 3	case 4	case 5	case 6	case 7
Stocks	50%	25%	25%	33%	80%	10%	10%
Bonds	25%	50%	25%	33%	10%	80%	10%
TIPS	25%	25%	50%	33%	10%	10%	80%

Exhibit 14: Percentage of policy-holders exercising surrender options for an allocation represented by case 1

	min	10%	20%	30%	40%	median	60%	70%	80%	90%	max
t = 0	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
t = 1	88.05%	96.41%	97.08%	97.52%	97.84%	98.11%	98.35%	98.58%	98.83%	99.09%	99.81%
t = 2	32.20%	88.20%	92.99%	94.99%	96.10%	96.86%	97.44%	97.90%	98.33%	98.78%	99.71%
t = 3	6.44%	62.73%	86.28%	92.52%	94.90%	96.19%	97.02%	97.65%	98.15%	98.68%	99.70%
t = 4	1.29%	13.48%	74.48%	90.20%	94.06%	95.75%	96.82%	97.51%	98.08%	98.63%	99.70%
t = 5	0.26%	2.70%	37.82%	88.21%	93.46%	95.52%	96.68%	97.44%	98.03%	98.60%	99.70%
t = 6	0.05%	0.54%	7.64%	86.51%	93.02%	95.34%	96.61%	97.39%	98.01%	98.59%	99.70%
t = 7	0.01%	0.11%	1.53%	84.80%	92.75%	95.23%	96.58%	97.36%	98.00%	98.58%	99.70%

of this study is not to analyse the advanced techniques of non-convex optimisation, we choose to discuss below seven arbitrary allocation strategies (see exhibit 13), and to gauge the impact of the inclusion of hedge funds.

For the 10,000 stochastic scenarios generated (see previous section), exhibit 14 shows the percentages of policy-holders not having exercised their surrender rights at each date, and this for an allocation equal to 50% in stocks, 25% in nominal bonds and 25% in TIPS (case 1).

We are now able to display (see exhibit 15) the results for each of the seven allocation strategies. As before, we examine a parsimonious model of the effect of including hedge funds. This inclusion leads to volatility reduction for the asset class (stocks or bonds) with which these hedge funds are associated. It thus appears that, for all seven strategies, the inclusion of hedge funds has a very significant impact on ALM risk, measured as the probability of a deficit (Prob(SF > 0)) and as an average deficit (expected SF). To assess the extreme risks of a mismatch between assets and liabilities resulting from the non-linear dependence of the liability value with interest rates we choose in addition to consider the impact on the probability of a deficit higher than 25% (rather than 10% as in the preceding sections), measured by Prob(SF > 25%).

The risk management benefits of including hedge funds are spectacular. For example, with an allocation of 25% to stocks, 50% to nominal bonds and 25% to TIPS (case 2), having hedge funds make up 15% of each class considered (in fact, only in the stocks and nominal bonds) leads to a fall of nearly 40% in the probability of a deficit (which falls from 15.85% to 11.45%) and of nearly 130% in the probability of an extreme deficit (which falls from 1.16% to 0.51%).

VI. Conclusion

Although most institutional investors are looking into hedge funds as a possible solution to the challenges posed by asset-liability management in the presence of serious concerns about the size of the equity and bond premium and the associated risks, very little is known about how to make these investment styles an integral part of asset-liability management.

This study provides evidence of the contribution of hedge funds in a surplus optimisation context. To this end, we have proposed a pragmatic approach that treats hedge funds not as an *addition* but as a *complement* to traditional asset classes (stocks and bonds), an approach that alleviates concerns over the modelling of hedge fund return distributions and parameter estimation. Our conclusion is that suitably designed hedge fund portfolios can be particularly attractive when expected returns must meet

Exhibit 15: Analysis of the net situation of an insurance company at the terminal date as a percentage of the initial value of contracts

case 1	*Allocation effective en HFs*	min	25%	median	75%	max	prob(SF > 0)	expected SF	prob(SF > 0,25)
0% HF	0%	– 72.71	1.19	4.89	8.39	38.20	17.37%	14.79	2.89%
5% HF	3.75%	– 90.37	1.36	4.93	8.25	32.41	15.95%	13.24	2.04%
15% HF	11.25%	– 60.46	1.98	5.04	8.04	29.78	12.42%	12.06	1.30%
25% HF	18.75%	– 57.32	2.61	5.05	7.56	21.74	9.31%	9.92	0.45%
35% HF	26.25%	– 39.24	2.99	5.11	7.22	19.04	6.27%	9.13	0.24%
case 2		min	25%	median	75%	max	prob(SF > 0)	expected SF	prob(SF > 0.25)
0% HF	0%	– 66.50	1.07	3.79	6.21	25.55	15.85%	11.11	1.16%
5% HF	3.75%	– 67.31	1.23	3.76	6.20	19.58	14.51%	10.15	0.80%
15% HF	11.25%	– 59.41	1.62	3.83	5.91	20.38	11.45%	9.50	0.51%
25% HF	18.75%	– 40.19	1.81	3.78	5.65	16.12	9.10%	7.93	0.15%
35% HF	26.25%	– 33.40	2.09	3.87	5.53	14.85	7.18%	7.58	0.11%
case 3		min	25%	median	75%	max	prob(SF > 0)	expected SF	prob(SF > 0.25)
0% HF	0%	– 52.38	0.99	3.48	5.63	21.49	15.49%	10.11	0.86%
5% HF	2.50	– 59.63	1.11	3.40	5.61	18.03	14.22%	9.30	0.59%
15% HF	7.50%	– 57.22	1.33	3.45	5.37	18.94	12.08%	8.82	0.33%
25% HF	12.50%	– 34.97	1.49	3.39	5.21	14.27	10.76%	7.61	0.12%
35% HF	17.50%	– 37.53	1.68	3.46	5.10	13.41	8.96%	7.56	0.17%
case 4		min	25%	median	75%	max	prob(SF > 0)	expected SF	prob(SF > 0.25)
0% HF	0%	– 63.26	1.23	4.10	6.72	26.32	15.22%	11.66	1.36%
5% HF	3.33%	– 72.47	1.37	4.08	6.64	21.54	13.95%	10.42	0.84%
15% HF	10%	– 59.12	1.82	4.14	6.37	20.90	11.22%	9.79	0.58%
25% HF	16.67%	– 42.12	2.06	4.11	6.07	15.98	8.70%	8.05	0.24%
35% HF	23.33%	– 33.74	2.37	4.17	5.90	14.92	6.55%	7.79	0.14%
case 5		min	25%	median	75%	max	prob(SF > 0)	expected SF	prob(SF > 0.25)
0% HF	0%	– 142.94	0.21	5.93	11.71	66.20	23.66%	22.46	8.40%
5% HF	4.50%	– 150.61	0.55	6.06	11.62	60.32	21.96%	20.91	7.12%
15% HF	13.50%	– 105.83	1.51	6.38	11.22	50.72	17.72%	17.77	4.35%
25% HF	22.50%	– 135.25	2.67	6.58	10.58	36.23	12.79%	15.58	2.41%
35% HF	31.50%	– 72.28	3.56	6.68	9.93	32.25	8.27%	14.01	1.19%
case 6		min	25%	median	75%	max	prob(SF > 0)	expected SF	prob(SF > 0.25)
0% HF	0%	– 75.61	0.29	2.98	5.48	24.04	21.99%	11.63	1.82%
5% HF	4.50%	– 57.95	0.46	3.02	5.47	19.35	20.09%	10.89	1.51%
15% HF	13.50%	– 58.99	0.82	3.14	5.30	19.42	16.25%	9.77	0.77%
25% HF	22.50%	– 42.28	1.03	3.07	5.02	16.66	14.14%	8.53	0.33%
35% HF	31.50%	– 42.94	1.29	3.23	4.97	15.42	11.86%	8.06	0.24%
case 7		min	25%	median	75%	max	prob(SF > 0)	expected SF	Prob(SF > 0.25)
0% HF	0%	– 50.92	0.18	2.05	3.98	15.25	22.23%	9.36	0.86%
5% HF	1%	– 44.59	0.15	2.03	3.92	15.12	22.64%	8.95	0.68%
15% HF	3%	– 53.73	0.17	1.99	3.87	16.37	22.10%	8.65	0.56%
25% HF	5%	– 37.70	0.20	1.97	3.81	13.55	21.77%	8.78	0.53%
35% HF	7%	– 46.26	0.20	1.99	3.86	12.25	21.65%	8.74	0.53%

For cases 1-7, various levels of diversification were tested corresponding to levels of allocation in hedge funds. The first column indicates the amount of hedge funds added to traditional asset classes (stocks and nominal bonds), while the second column indicates the actual allocation in hedge funds as part of the total allocation. For case 1, for example, the line 15 HF% translates into an allocation of 42.5% in stocks (50%*(100%-15%)), 21.25% in nominal bonds(25%*(100%-15%)), 25% in TIPS (not-diversified class) and 11.25% in hedge funds (15%*(50%+25%)). From 10,000 random trajectories, the Prob(SF > 0) gives the proportion of trajectories posting a deficit at the final date. The expected SF indicator corresponds to the average loss of all the trajectories posting a deficit at the final date. Lastly, Prob(SF > 0.25) represents the proportion of trajectories posting a deficit higher than 25% of the initially paid sum at the final date.

the objective of offsetting liabilities. The appeal of these portfolios stems from the diversification properties of hedge funds and from their positive impact on the tail-distribution and extreme risks of stock and bond portfolios. ■

1. It has been shown that financial products which offer non-linear return profiles are particularly useful in ALM (see Draper and Shimko 1993).
2. Finally, and for the sake of completeness, it is appropriate to mention non-linear risk-profiling management techniques, the goal of which is to provide a compromise between risk- and return-free approaches on the one hand, and risky approaches that do not allow the liability constraints to be guaranteed on the other (see Leibowitz and Weinberger 1982 for the *contingent optimisation* technique or Amenc, Malaise, and Martellini (2004) for a generalisation in terms of a *dynamic core-satellite* approach).
3. Given that our models are based on Gaussian distributions, they do not allow us to incoporate higher moments. Allowing for models with jumps and or stochastic volatility would allow us to fit skewness and kurtosis to stock and bond distributions. This, however, would come at the cost of parsimony, which would make our results less robust.
4. For stocks we have used data on world markets (see table 34-1, page 311). Note that the long-term volatility estimate is significantly higher than current values. For bonds, because of the impact of a high inflation period in some European countries in the vicinity of World War II, we have focused on the US estimate (see table 6-1, page 79), and we have added a 0.4% credit spread to the 2.1% real rate plus a 3.3% inflation estimate. For the return on TIPS, we have used the US inflation rate (3.3%) plus the real short-term rate (1%), in the absence of a reliable estimate of the long-term risk premium for that asset class.
5. It should be noted that the two approaches are not completely opposed. A combination of both approaches can be achieved, by initially carrying out the RBS factors approach and by inferring ABS factors from RBS factors (see Fung and Hsieh 2002).
6. The condition that an increase in portfolio skewness follows from a third moment beta lower than 1 is valid only in the event that the skewness of the portfolio is negative. When the skewness of the portfolio is positive, then the condition is that the third moment beta is greater than, as opposed to less than, one.
7. Analyses conducted on sample extended to more recent periods make it possible to obtain similar results.
8. This is consistent with the fact that even moments (variance and kurtosis) are natural measures of dispersion (*i.e.*, risk), while odd moments (expected return, skewness) are measures of location, which are notoriously less stable.
9. Martellini, Vaissié, and Ziemann (2005) analyse the robustness through time of co-moment estimates on the basis of an analysis of time-conditional properties of higher moment beta, in which these coefficients are modelled by a Kalman smoother technique.
10. It should be noted that for simplicity our models do not explicitly incorporate a mean-reverting component. Introducing this element would further magnify the difference between short-term and long-term horizons. We leave this for furthe research.
11. In what follows, we assume away the complexity related to the accounting and solvency impacts of various asset allocation decisions, to focus on the purely financial aspects.
12. Not considering these reserves obviates the problems of financing the solvency margin and the possibility of bankruptcy during simulation. The bankruptcy will be declared if it takes place on the final date (t = 8).

Appendix A: Construction Methodology for the EDHEC Alternative Indices

The difficulties of developing indices, clear enough in the traditional universe, are exacerbated in the alternative investment world. Because of the scarcity of information, it is difficult to respect the logic of representativeness through market capitalisation. Finding a benchmark that is representative of a particular management universe is therefore not a trivial problem. As it is impossible to come up with an objective judgment on the best existing index, it would be natural to use some combination of competing indices (*i.e.*, the indices representative of a given investment style available on the market) to achieve a better understanding of the common information about a particular investment style.

One straightforward method would involve computing an equally-weighted portfolio of all competing indices. Since competing hedge fund indices are based on different sets of hedge funds, the resulting portfolio of indices would be more exhaustive than any of the competing indices it is drawn from. This logic, however, might be taken one step farther.

An optimal solution involves using factor analysis techniques to generate a set of alternative indices that can be thought of as the best possible one-dimensional summaries of information conveyed by competing indices for a given style, in the sense of the largest fraction of the variance explained. Technically, this amounts to using the first component of a principal component analysis of competing indices (5 to 9 depending on the strategy). The composition of the series of EDHEC indices of indices is obtained through a simple normalisation of the weights of the first component (see Amenc et al. 2004for further information). The EDHEC indices are rebalanced every three months using a 36-month rolling window calibration period.

Appendix B: Use of Hedge Funds by Institutional Investors

In 2005, EDHEC surveyed 1,000 top institutional investors. More than 150 responded, providing us with a window onto their practices in terms of alternative investment. The sample included all categories of institutional investors, *i.e.*, pension funds, investment banks, insurance companies, foundations and associations, industrial and commercial companies. The results showed that 51% of institutional investors used hedge funds and that hedge funds accounted for anywhere from 0.1% to 20%. (for an average of 7%) of their investments.

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