Assessing Extreme Risk-adjusted performance of Hedge Funds

EMMANUELLE FROMONT*
Senior lecturer
Université
Rennes 1

Hedge Fund performance is a crucial parameter not only for investors who build their portfolios but also for fund managers who must justify skill fees they receive. The problem is that performance estimation of hedge funds is a complex task. As for appreciation of their underlying risk, it isn't possible to work with indicators used for mutual funds which don't take the specificities and complexity of hedge fund strategies into account. One advantage often attributed to hedge funds is their absolute performance. They are supposed to offer a regular performance whatever the evolution of financial markets. To emphasise this aspect of hedge fund performance, fund managers and financial press mostly use the Sharpe ratio. But this indicator doesn't make it possible to consider extreme risk of hedge funds which is judged as particularly significant and alarming (European central Bank's financial stability report 2006). The Sharpe ratio is all the more unsuitable as investors are very sensitive to the occurrence of substantial losses (Scott and Horvath [1980]; Pratt and Zeckhauser [1987]). As a consequence, the question of the accuracy of investment decisions made by investors must be asked since hedge fund industry shows a significant and fast growth. As no specific regulation for hedge funds exists, it is essential that an extreme risk-adjusted performance indicator is used by financial actors and institutional investors. Indeed, the latter promise their customers not only to reach a performance target but also to use relevant tools.

Fromont [2005; 2007] emphasises *Extreme Value Theory* contributions to analyse and quantify extreme risk of hedge funds. These researches disclose that the Value-at-Risk estimated from Generalised Pareto Distribution fitting to funds' extreme losses (VaR_{EVT}) is a relevant risk measure in comparison with usual risk indicators.

The purpose of this paper is to show how *Extreme Value Theory (EVT)* can be also useful to appreciate performance of hedge funds. More exactly, we propose a new absolute performance indicator which includes the VaR_{EVT} to take their extreme risk into account. This indicator makes it possible to penalise hedge funds having a high probability of extreme losses. In this way, investors can make investment decisions more compatible with their requirements. Moreover, empirical evidence shows how the mathematical Extreme Value Theory offers some interesting and original perspectives to solve problems in the performance evaluation of financial products having a leptokurtic and asymmetric return distribution.

The first part of this paper explains reasons for adjusting performance of hedge funds by their underlying extreme risk. We also suggest a new performance indicator named "Extreme Sharpe ratio" which bypasses limits of traditional indicators. Results of empirical study are presented in the second part. They tend to emphasize that unlike the Extreme Sharpe Ratio, usual performance measures lead to overestimate the hedge fund performance and their persistence.

I. Extreme Sharpe Ratio: an answer to investors' extreme risk aversion

I.1. THE SHARPE RATIO: DANGERS OF ITS HEGEMONY

As suggested in the modern portfolio theory, the performance of a fund must be assessed by taking their risk into account. The purpose is to penalise investments with a too low return per unit of risk. The most famous indicator is the Sharpe ratio. Its popularity is essentially due to its easy implementation. Proposed by William F. Sharpe [1966], this ratio is defined as the excess return over the risk-free rate divided by the standard deviation of returns. The higher the ratio Sharpe is, the more profitable the fund is. However, when the ratio is negative it means that it yields less than a free-risk asset.

The Sharpe ratio is the most widely used measure to assess hedge fund performance (Edhec Report [2004]). Nevertheless its simplicity hides some conceptual limits that are incompatible with hedge fund characteristics. Standard deviation of returns doesn't make it possible to consider their extreme risk. However, many studies

* Emmanuelle.fromont@univ-rennes1.fr

show that the latter is particularly significant in hedge fund industry (Agarwal and Naik [2000c], Liang [2000], Amin and Kat [2001], Mitchell and Pluvino [2001] and Fromont [2005; 2006]). In such a case, investors run the risk of overestimating hedge fund performance and selecting the least profitable funds per global risk unit. When markets are very volatile for example, hedge funds tend to exhibit a return and a standard deviation respectively higher and lower than traditional assets. Hedge funds therefore appear more profitable than market indexes. By using the Sharpe ratio, investors could wrongly substitute traditional assets for hedge funds in their portfolios. Moreover, this performance measure can lead investors to choose the lowest profitable funds given their global risk. According to Kat [2003], this scenario is all the more likely because funds with the best Sharpe ratios often show the highest skewness and kurtosis. He claims that no free lunch would exist in the hedge fund world: a better return is usually associated with a higher risk. Lastly, it is dangerous to consider that two funds with the same Sharpe ratio are substitutable. If the first fund shows a higher extreme risk, these investments can't be considered like equivalent. Indeed, the second one is the most profitable.

I.2. AN ALTERNATIVE MEASURE: THE EXTREME SHARPE RATIO

The originality of the performance indicator we propose is to introduce the Value-at-Risk estimated from the Extreme Value Theory (VaR_{EVT}) to take the extreme risk of funds into account. The purpose is to penalise investments having a high extreme risk because investors exhibit a strong aversion to heavy losses. Named Extreme Sharpe ratio, it is defined by the following expression:

$$Extreme\_Sharpe\ ratio = \frac{R - MAR}{VaR_{EVT}}$$

R is the annualised return, MAR corresponds to the Minimum Acceptable rate of Return and the VaR_{EVT} represents the extreme risk estimated from the *Extreme Value Theory*.

Based on Pickands' theorem [1975], the VaR_{EVT} is assessed by the equation:

$$VaR_{EVT_n} = u_n + \frac{\hat{\sigma}_n}{\hat{\xi}_n}\left(\left(\frac{n(1-p_n)}{N_u}\right)^{-\hat{\xi}_n} - 1\right)$$

where $\hat{\sigma}_n$ and $\hat{\xi}_n$ represent the parameter estimators of Generalised Pareto Distribution (GPD) fitting to N_u extreme losses situated beyond a threshold u which marks the beginning of the left tail of the return distribution.

Values of the Extreme Sharpe ratio belong to $[-\infty; +\infty]$. The numerator corresponds to the excess return over the investor's Minimum Acceptable rate of Return and the denominator represents the portfolio's extreme risk. The VaR_{EVT} is defined like the maximum potential loss for a given confidence interval. Estimated from the extreme asymptotic distribution (Generalised Pareto Distribution), this risk measure makes it possible to do extrapolations beyond the biggest historical losses of the portfolio. Fromont [2007] demonstrates that the VaR_{EVT} is a reliable measure to assess the risk of hedge funds which show significant extreme losses. The higher the extreme Sharpe ratio is, the more profitable the investment is. However, a negative ratio means that the return of portfolio doesn't reach the investor's return expectation. In such a case, the portfolio is not compatible with the investor's acceptable return even if its extreme risk is low.

It is important to notice that the use of the Extreme Sharpe ratio requires long-enough data series. For example, our empirical study shows it is not possible to evaluate GPD parameters with 60 daily data.

The Extreme Sharpe ratio is an "absolute" performance measure. Unlike relative performance indicators, the Extreme Sharpe ratio doesn't refer to any benchmark. It takes investors' expectations into account by including their Minimum Acceptable rate of Return (MAR) which is described by the Fishburn's normative utility function [1977]. In the prospect theory, Kahneman and Tversky [1979] also highlight the influence of this minimum return on investors' decisions. They claim that investors tend to take risks only when the return is below this threshold. According to Amenc and al. [2004], the MAR level must be accurately chosen to avoid an overestimation of the performance. Backmann and Scholz [2003] advise to fix it between 0% and the risk-free rate. According to Keating and Shadwick [2002], some hedge fund contractual specificities such as the lockup period, the hurdle rate and the high watermark must be used to determine this minimum return.

In any case, the MAR must be compatible with investors' preferences. So, it could be judicious to employ a function which determines the performance for different MAR levels. This proposition enables investors to choose the most compatible situation given their return expectations.

II. The absolute performance estimation of hedge fund strategies

II.1. DESCRIPTION OF STUDY

a. Data

Our study concerns daily returns of four hedge fund indexes supplied by Standards & Poors between October 2002 (date of launch) and January 2006. Today, S&P is the only data supplier offering hedge fund return series which are long enough to use the Extreme Value Theory. These four indexes describe returns of the three main categories of hedge fund strategies (arbitrage strategies, Event-Driven strategies and Directional/Tactical strategies) and returns of a global hedge fund index representing the whole hedge fund strategies.

Arbitrage strategies try exploiting market inefficiencies. They are characterised by a low systematic exposure to the market. With a medium exposure, Event-Driven Strategies endeavour to exploit events such as merger and bankruptcy announcements but also debt rescheduling declarations. Lastly, Directional/Tactical strategies appear very exposed to the market by attempting to profit from some large trends on stocks, bonds, currencies, interest rates and commodities. To complete the analysis, the S&P 500 performance is also studied. So, the performances of hedge fund strategies are compared with one another and with the US market index.

b. Definition of performance measures

In this comparative study, four performance measures are used. Firstly, the Extreme Sharpe ratio is the new indicator we propose to take extreme risk of hedge funds into account. This performance measure is adjusted by the VaR_{EVT} evaluated from the Extreme Value Theory. The second performance measure is the Sharpe ratio which is based on the Gaussian distribution hypothesis. The two other performance indicators were created to improve performance evaluation of financial assets with Non-Normal return distribution. Sortino ratio is the return excess over the Minimum Acceptable rate of Return (MAR) adjusted by the downside risk which occurs when return is below the MAR:

$$\begin{aligned} Ratio\ de\ Sortino &= \frac{E(R)-RMA}{Semi\ Variance} \\ &= \frac{E(R)-RMA}{\sqrt{\frac{1}{N}\sum_{t=1}^{T}(R_t-RMA)^2}} \\ &avec\ R_t < RMA \end{aligned}$$

R_t corresponds to the return of the period t, MAR is the investor's Minimum Acceptable rate of Return and T is the number of months during which return is below the RAM.

The latest indicator was proposed by Gregoriou and Gueyie [2003]. The Modified Sharpe ratio is characterized by the use of Cornish Fisher's modified VaR (VaR_{CF}). Contrary to the Normal VaR, the originality of the VaR_{CF} is to introduce skewness and kurtosis.

$$\text{Modified Sharpe Ratio} = \frac{\mu - Rf}{VaR_{CF}}$$

with

$$\begin{aligned} VaR_{CF} = \mu - \sigma\Big[Z_c + \frac{1}{6}(Z_c^2-1)S + \frac{1}{24}(Z_c^3-3Z_c) \\ K - \frac{1}{36}(2Z_c^3-5Z_c)S^2\Big] \end{aligned}$$

μ corresponds to the annualised return of the portfolio, σ is the annualised deviation standard of returns, S and K are respectively the skewness and the excess of kurtosis, R_f is the risk-free rate and Z_c represents the critical value associated to the normal law given a probability level α.

II.2. RANKING OF MAIN HEDGE FUND STRATEGIES

a. The influence of the investor's expected return and the VaR_{EVT}'s confidence level

The table 1 presents annualised return and standard deviation of each index. Over the study period, the annualised return of S&P500 is better than the S&P hedge funds indexes. Indeed, the S&P 500 return (16.87%) is respectively 1.5 times, 2 times and nearly 6 times higher than the Event Driven HF index (hfED), the Arbitrage HF Index and the Arbitrage HF index (hfA).

However, this favourable position of the market index must be qualified with its standard deviation. Indeed, the annualised volatility of S&P500 appears 3 to 8 times higher than the hedge fund indexes. However, as previously underlined, volatility measure doesn't make it possible to consider the extreme risk of hedge funds. In such a case, we suggest evaluating performance of the main alternative strategies thanks to the Extreme Sharpe ratio. By using this performance measure, indexes having the highest daily potential losses will be penalised. Moreover, the Extreme Sharpe ratio offers the advantage to identify investments that don't reach the investor's minimum expected return.

The performance appreciation obtained with the Extreme Sharpe ratio is presented in the table 2. Values are estimated with several VaR_{EVT} levels (95%, 99% and 99.50%) and MAR (from 0% to 10%). We can notice that values of the three HF indexes become negative beyond a MAR level. That occurs from a 2.5% threshold for the Arbitrage index (hfA), from 7% for the Global HF index (hfI) and from 7.5% for the Directional/ Tactical index (hfDT). In such cases, their return isn't high enough to reach the minimum expected return.

As shown in the Graphics 1 and 2, performance evaluation from the Extreme Sharpe ratio is a linear function of the MAR level. So, the index ranking depends on the investor's return expectation. For example, with a 6% MAR, the Directional/ Tactical strategies (hfDT) are less profitable than the global HF index (hfI) whereas we obtain a reverse ranking with a 6.5% MAR. It emphasises the importance to accurately define this parameter.

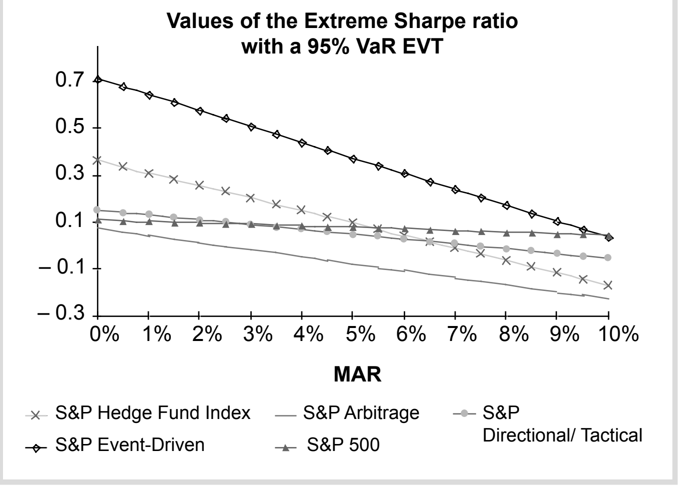
Table 1. Annualised return and annualised volatility of Hedge Funds S&P indexes and the S&P 500 index (period: October 2002- January 2006)

	S&P Hedge Fund Index	S&P Arbitrage Index	S&P Directional/ Tactical Index	S&P Event-Driven Index	S&P 500
Flat return	16.51	5.73	17.88	26.13	44.28
Annualized return	6.83	2.44	7.37	10.56	16.84
Annualized volatility	2.50	4.17	5.82	2.34	19.11

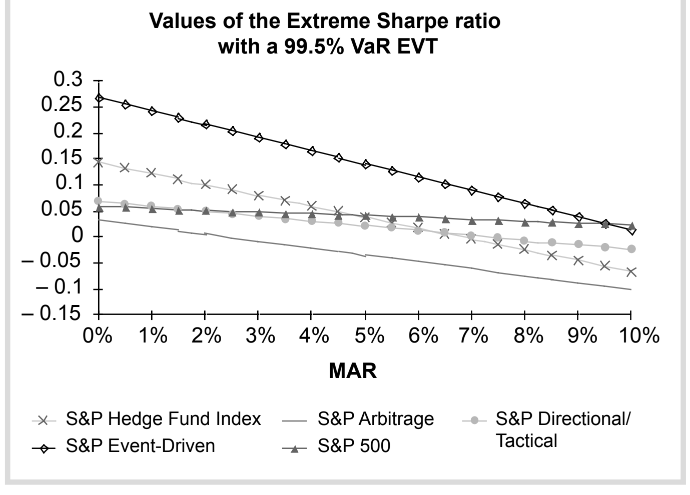
Table 2. Values of the Extreme Sharpe ratio estimated with various levels of Extreme VaR (95%, 99% and 99.50%) for several levels of Minimum acceptable rate of return (from 0% to 10%).

	S&P Hedge Fund Index			S&P Arbitrage Index			S&P Directional/ Tactical Index			S&P Event-Driven Index			S&P 500		
Minimum Acceptable Return / Prob VaR	95.00%	99.00%	99.50%	95.00%	99.00%	99.50%	95.00%	99.00%	99.50%	95.00%	99.00%	99.50%	95.00%	99.00%	99.50%
0.0%	0.364	0.190	0.143	0.073	0.042	0.033	0.152	0.087	0.068	0.712	0.360	0.267	0.113	0.071	0.058
0.5%	0.337	0.176	0.133	0.058	0.033	0.026	0.142	0.081	0.063	0.679	0.343	0.255	0.109	0.069	0.057
1.0%	0.310	0.162	0.122	0.043	0.025	0.019	0.131	0.075	0.059	0.645	0.326	0.242	0.106	0.067	0.055
1.5%	0.284	0.148	0.112	0.028	0.016	0.013	0.121	0.069	0.054	0.611	0.309	0.229	0.103	0.065	0.053
2.0%	0.257	0.134	0.101	0.013	0.007	0.006	0.111	0.063	0.050	0.578	0.292	0.217	0.099	0.063	0.052
2.5%	0.231	0.120	0.091	**-0.002**	**-0.001**	**-0.001**	0.101	0.057	0.045	0.544	0.275	0.204	0.096	0.061	0.050
3.0%	0.204	0.107	0.080	**-0.017**	**-0.010**	**-0.008**	0.090	0.051	0.040	0.510	0.258	0.191	0.093	0.059	0.048
3.5%	0.177	0.093	0.070	**-0.032**	**-0.018**	**-0.014**	0.080	0.046	0.036	0.476	0.241	0.179	0.089	0.056	0.046
4.0%	0.151	0.079	0.059	**-0.047**	**-0.027**	**-0.021**	0.070	0.040	0.031	0.443	0.224	0.166	0.086	0.054	0.045
4.5%	0.124	0.065	0.049	**-0.062**	**-0.035**	**-0.028**	0.059	0.034	0.026	0.409	0.207	0.154	0.083	0.052	0.043
5.0%	0.097	0.051	0.038	**-0.077**	**-0.044**	**-0.034**	0.049	0.028	0.022	0.375	0.190	0.141	0.079	0.050	0.041
5.5%	0.071	0.037	0.028	**-0.092**	**-0.052**	**-0.041**	0.039	0.022	0.017	0.341	0.173	0.128	0.076	0.048	0.039
6.0%	0.044	0.023	0.017	**-0.107**	**-0.061**	**-0.048**	0.028	0.016	0.013	0.308	0.156	0.116	0.073	0.046	0.038
6.5%	0.018	0.009	0.007	**-0.122**	**-0.070**	**-0.055**	0.018	0.010	0.008	0.274	0.139	0.103	0.069	0.044	0.036
7.0%	**-0.009**	**-0.005**	**-0.004**	**-0.137**	**-0.078**	**-0.061**	0.008	0.004	0.003	0.240	0.122	0.090	0.066	0.042	0.034
7.5%	**-0.036**	**-0.019**	**-0.014**	**-0.152**	**-0.087**	**-0.068**	**-0.003**	**-0.001**	**-0.001**	0.207	0.104	0.078	0.063	0.040	0.032
8.0%	**-0.062**	**-0.032**	**-0.024**	**-0.167**	**-0.095**	**-0.075**	**-0.013**	**-0.007**	**-0.006**	0.173	0.087	0.065	0.059	0.037	0.031
8.5%	**-0.089**	**-0.046**	**-0.035**	**-0.182**	**-0.104**	**-0.082**	**-0.023**	**-0.013**	**-0.010**	0.139	0.070	0.052	0.056	0.035	0.029
9.0%	**-0.115**	**-0.060**	**-0.045**	**-0.197**	**-0.112**	**-0.088**	**-0.034**	**-0.019**	**-0.015**	0.105	0.053	0.040	0.053	0.033	0.027
9.5%	**-0.142**	**-0.074**	**-0.056**	**-0.212**	**-0.121**	**-0.095**	**-0.044**	**-0.025**	**-0.020**	0.072	0.036	0.027	0.049	0.031	0.025
10%	**-0.169**	**-0.088**	**-0.066**	**-0.227**	**-0.129**	**-0.102**	**-0.054**	**-0.031**	**-0.024**	0.038	0.019	0.014	0.046	0.029	0.024

Graph 1. The Extreme-Sharpe ratio of the hedge fund S&P indexes and the S&P 500 index with a 95% VaR_{EVT} and different levels of MAR



Graph 2. The Extreme-Sharpe ratio of the hedge fund S&P indexes and the S&P 500 index with a 99.5% VaR_{EVT} and different levels of MAR



Whatever the index, the Extreme Sharpe ratio decreases when the investor's expected return increases. However, this phenomenon is more marked with HF indexes (especially with the Event Driven index and the Global HF index) than with the market index. Their excess return per unit of extreme risk decreases stronger than the S&P500. When the MAR increases, the HF indexes are more penalised by their extreme risk. That's the reason why the S&P500 gets the top position with a 10% MAR whereas it is less well classified than the three hedge fund indexes with a MAR from 0% to 2.5% (with a 95% VaR_{EVT}).

Moreover, empirical evidence highlights the influence of the VaR_{EVT}'s confidence interval on results. With a 99.5% VaR_{EVT} for example, the S&P 500 becomes more profitable than Directional/ Tactical index (hfDT) from a 2% MAR whereas this threshold is 3% with a 95% VaR_{EVT}. The higher the VaR_{EVT}'s confidence interval is, the less favourable the relative position of the riskiest indexes is. To avoid overestimating hedge fund performance, investors must therefore attach importance to the choice of confidence level used to assess the VaR_{EVT}. Investors with a strong aversion to the extreme losses must use a high confidence interval.

Now, let's compare these results with those obtained from three other performance measures which are commonly employed to study hedge funds.

b. The risk of overestimation associated with usual ratios

The risk measure used in the Modified Sharpe ratio (Cornish Fisher VaR) is evaluated with three confidence intervals (95%, 99% and 99.5%). The Sharpe and the Modified Sharpe ratios are estimated for different risk-free rates from 0% to 5%. As for the Sortino ratio, it is assessed with MAR from 0% to 10%.

Let's compare the performance rankings got with the Extreme Sharpe ratio and the other performance measures. Beyond a 5% MAR, the index rankings are the same with the Extreme Sharpe ratio and the Sortino ratio. As regards rates from 5% to 2%, we can notice some ranking differences between our measure and the other ratios (Graphics 3). With a 5% rate for example, the 99.5% Extreme Sharpe ratio ranks the global HF index at the third place whereas it is rated second with the other measures. In the same way with a 2.5% rate, Directional/Tactical strategies appear less profitable than the S&P500 when the Extreme Sharpe ratio is used whereas we get a reverse ranking with the three other indicators. These results suggest the Extreme Sharpe ratio penalises heavier some hedge fund indexes. Conversely, the Sharpe, Sortino and Modified Sharpe ratios don't make it possible to disadvantage investments having a low excess return per extreme risk unit. Fromont [2007] shows that unlike the VaR_{EVT}, the deviation standard, the semi-deviation and the CF VaR are not reliable to assess the extreme risk of hedge funds. As a consequence, the usual performance measures lead investors to overestimate hedge funds. So, the Extreme Sharpe ratio seems to be more appropriate to extreme risk aversion investors. As for the choice of VaR_{EVT} confidence level, it must be high enough to include the highest losses. With 5% and 2% rates for example, the performance of Directional/Tactical strategies is penalised heavier by the 99.5% Extreme Sharpe than by the 99% Extreme Sharpe ratio and the usual performance indicators.

II.3. PERFORMANCE PERSISTENCE OF ALTERNATIVE STRATEGIES

Given investors are not allowed to sell their shares for a period of time determined by hedge fund contracts (lockup period), investors need to assess if the fund performance tends to persist through time. It is necessary to evaluate the nature of this persistence: a short term or a long term phenomenon. Thanks to this estimation, the investor has the possibility to judge the soundness of performance fees received by hedge fund managers.

In studies of hedge fund performance persistence, two main categories of indicators are generally used. The first one is composed of parametric indicators. It consists in the regression of the performance for a period (N) on the performance of the previous period (N-1). This method is employed in Brown, Goetzmann and Ibbotson [1999], Naik and Agarwal [2000b], Caglayan and Edwards [2001], Brorsen and Harri [2002], Kat and Menexe [2003] and De Souza and Gokcan [2004]. With this approach, the underlying distribution is supposed to be known and the relationship between variables (N and N – 1) is linear. To avoid these limits, some authors advise to use non-parametric estimators.

a. A Non-parametric approach to assess the funds' performance persistence

The non-parametric estimation of performance persistence consists in classifying the performance of funds on several successive periods. Then, the ranking persistence is evaluated with the Chi² test and the Kolmogorov-Smirnov multi-period test.

The Chi² test is employed to measure the performance independence between two successive periods. Firstly, portfolios are divided into two groups: the "winners" and the losers. Winners (W) show a higher performance than the population median. Conversely, losers (L) have a lower performance. Then, the ranking of two successive periods are compared. The performance is persistent if the portfolio belongs to the same group in the two successive periods (WW or LL). If the portfolio is a Winner (Loser) in the first period and a Loser (Winner) in the second one, its performance is non persistence. The Chi² test consists in comparing the theoretical frequency of four groups (WW, LL, WL and LW) to their empirical frequency by using the following statistic:

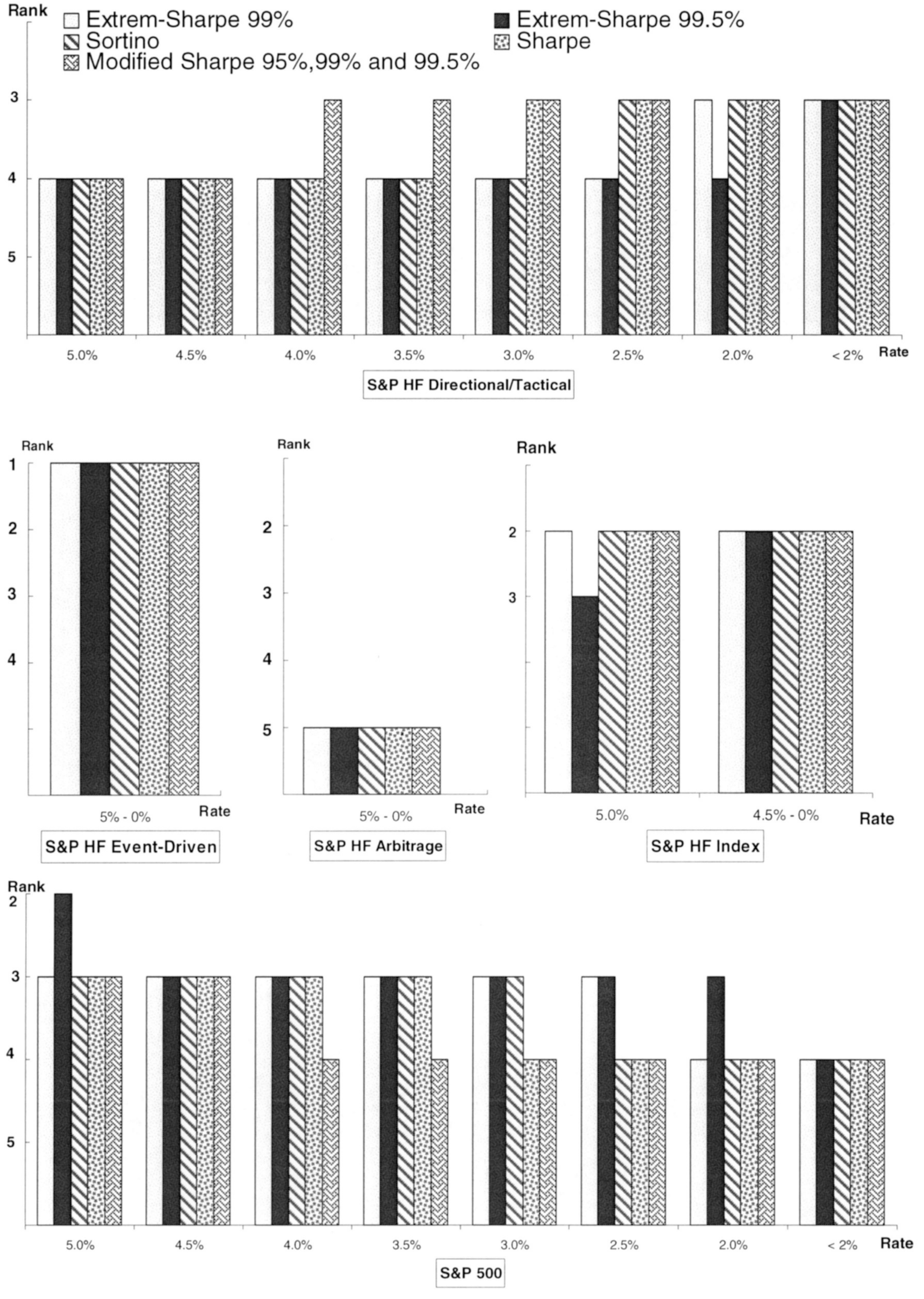
$$\chi^2 = \sum_{i=1}^{4} \frac{O_i - E_i}{E_i}$$

O_i represents the number of observations belonging to each category i (WW, LL, WL, LW) and E_i is their theoretical number.

The performance independence is rejected when the statistic is higher than the critical value associated to the Chi² distribution for 1 degree of freedom. This test is used to evaluate the hedge fund performance persistence in Park and Staum [1998], Naik and Agarwal [2000b], Kouwenberg [2003] and also Koh, Koh and Teo [2003].

The Kolmogorov-Smirnov test makes it possible to assess the performance persistence over several successive periods. Agarwal and Naik [2000b] highlight

Graph 3. The S&P index ranking obtained with different performance measures and MAR or risk-free rates (on the abscissa). From the rank 1 for the most profitable to the rank 5 for the least profitable



that this test is more robust because a succession of gains or losses is most unlikely to occur by chance. So, this test seems more suitable to judge managers' skills. The K-S test consists in comparing the adequacy between the empirical and theoretical distributions of successive gains and losses. Under the hypothesis Ho for example, the theoretical probability to be a winner or a loser during three (four) successive periods is equal to 1/8 (1/16). To test Ho, the Kolmogorov-Smirnov's statistic D is used. It is the biggest distance between the cumulated frequency of observations F(x) and the cumulated theoretical distribution G(x). Agarwal and Naik [2000b], DeGroot and Schervish [2002] and Koh, Koh and Teo [2003] advise using this test to assess the adequacy between the Normal distribution and the hedge fund performance distributions. Unlike the Chi² test, the Kolmogorov-Smirnov test gives more accurate estimations with small data samples such as hedge fund data series.

b. Hedge funds' performance: a short term phenomenon

Our persistence analysis concerns quarterly, half-yearly and annual performances. The purpose is to differentiate the short term and long term persistences. Five performance indicators are used: the Sharpe ratio, the Sortino ratio, the 99% modified Sharpe ratio and the 95% and 99% Extreme Sharpe ratios. Performance measures are calculated with a 3% MAR (or risk-free rate) which corresponds to the average interest rate during the period. Notice that the quarterly persistence can't be evaluated from the Extreme Sharpe ratio. Quarterly data series don't contain enough extreme values to assess parameters of the GPD distribution used to calculate the VaR_{EVT}. The table 3 presents Chi² statistics of each hedge fund index. They are calculated from four performance indicators and three time horizons (quarterly, half-yearly and annual).

Firstly, the probabilities to be Winner-Winner (WW), Loser-Loser (LL), Winner-Loser (WL) or Loser-Winner (LW) during two successive periods are evaluated. If these probabilities are identical, statistic is equal to 0. In such a case, no performance persistence exists. Probabilities associated with statistics are shown in italics.

On the whole, the quarterly performance of hedge fund indexes seems to be persistent. With a 5% threshold, the majority of statistics are significant. Only the performance persistent of the global HF index is rejected when the Sortino ratio is used: the probability associated with the statistic (10.2%) is higher than the 5% threshold.

Nevertheless, half-yearly performances of the global HF and Arbitrage indexes aren't persistent. Whatever the performance indicator, the performance independence of these indexes can't be rejected (threshold of 5%). Conversely, half-yearly performances of the Directional/Tactival and Event Driven indexes stay relatively stable. Only the Extreme Sharpe ratio with a 99% VaR_{EVT} leads to conclude to the non-persistence of half-yearly performance of the Directional/Tactical strategies. It means that the excess return per extreme risk unit between two consecutive semesters tends to be instable.

Concerning the long term performance persistence, conclusions of the Chi² test depend on the performance indicator. When Sharpe and Modified Sharpe ratios are employed, the annual performance of every

Table 3. The Hedge fund index Chi² values calculated with different performance indicators and time horizons (quarterly, half-yearly and annual)

	Chi²	Sharpe Ratio		Sortino Ratio		Modified Sharpe Ratio 95%		Extreme Sharpe ratio 95%		Extreme Sharpe ratio 99%	
three months	S&P Hedge Fund Index	**7.33**	*0.007*	2.67	*0.102*	**7.33**	*0.007*				
	S&P Arbitrage Index	**7.33**	*0.007*	**7.33**	*0.007*	**7.33**	*0.007*				
	S&P Directional/Tactical Index	**16.67**	*0.000*	**8.67**	*0.003*	**16.67**	*0.000*				
	S&P Event-Driven Index	**16.67**	*0.000*	**16.67**	*0.000*	**16.67**	*0.000*				
six months	S&P Hedge Fund Index	2.20	*0.138*	2.20	*0.138*	2.20	*0.138*	2.20	*0.138*	0.60	*0.439*
	S&P Arbitrage Index	3.80	*0.051*	3.80	*0.051*	3.80	*0.051*	3.80	*0.051*	3.80	*0.051*
	S&P Directional/Tactical Index	**8.60**	*0.003*	**8.60**	*0.003*	**8.60**	*0.003*	**8.60**	*0.003*	2.20	*0.138*
	S&P Event-Driven Index	**15.00**	*0.000*	**15.00**	*0.000*	**15.00**	*0.000*	**15.00**	*0.000*	**15.00**	*0.000*
one year	S&P Hedge Fund Index	**6.00**	*0.014*	2.00	*0.157*	**6.00**	*0.014*	2.00	*0.157*	2.00	*0.157*
	S&P Arbitrage Index	**6.00**	*0.014*	**6.00**	*0.014*	**6.00**	*0.014*	**6.00**	*0.014*	**6.00**	*0.014*
	S&P Directional/Tactical Index	**6.00**	*0.014*	2.00	*0.157*	**6.00**	*0.014*	2.00	*0.157*	2.00	*0.157*
	S&P Event-Driven Index	**6.00**	*0.014*	**6.00**	*0.014*	**6.00**	*0.014*	**6.00**	*0.014*	**6.00**	*0.014*

The probabilities of statistics are written in italics. The Modified Sharpe ratios are estimated with a 95% Cornish-Fisher's VaR. The Extreme Sharpe ratios are calculated with a 95% and 99% VaR EVT.

hedge fund index appears persistent. By using Sortino and Extreme Sharpe ratios, only Arbitrage and Event Driven strategies seem to offer annual performance persistence.

Now, let's analyse conclusions provided by the Kolmogorov-Smirnov test. By using this statistic, an index is unlikely to be a winner or a loser over several consecutive periods by chance. The Kolmogorov-Smirnov's statistics are evaluated by comparing the normal distribution with the empirical distributions associated to two or more consecutive gains (losses). For example, we evaluate whether the probability to be a winner (or a loser) during three, four and five successive periods is significantly different that respectively 1/8, 1/16 and 1/32. The table 4 shows results of the KS test. When statistics D are above the critical threshold (5%), they are written in bold and the persistence hypothesis is accepted. Values in brackets indicate the maximum number of successive periods during which the index is a winner or a loser. With the Sortino ratio for example, Directional/Tactical strategies show a loser position during 6 consecutive quarters whereas they don't succeed in having a winner position during more than one quarter.

It appears that all hedge fund indexes have a significant persistence of their quarterly performance. Whatever the performance indicator, conclusions are similar. The global hedge fund and Event Driven indexes tend to stay winners. As for the Arbitrage and Directional/Tactical indexes, they maintain their loser position. With a winner position during 7 successive quarters, Event Driven strategies seem to be the most profitable. Conversely, the worst performance is shown by Directional/Tactical strategies. These strategies keep a loser position during 6 to 10 successive quarters.

Now, let's focus on results concerning the half-yearly performance persistence. Firstly, we can notice that Event Driven strategies have significant half-yearly performance persistence. They maintain a winner position during 6 half-years. Conversely, half-yearly performance of the global hedge fund index doesn't appear persistent: all probabilities are higher than 5%. As for Arbitrage and Directional/Tactical strategies, their bad performance is confirmed on a half-yearly horizon: they are classified like loser during 3 or 5 successive half-years. Except when the 99% Extreme Sharpe ratio is used, the loss series of the Directional/Tactical Index are significantly persistent with a 5% critical threshold.

It's interesting to notice the influence of the heaviest losses on results when the Extreme Sharpe ratio is used. Indeed, it appears that the half-yearly performance of Directional/Tactical index becomes non persistent when the VaR_{EVT} confidence level increases from 95% to 99%. To conclude about the half-yearly performance of alternative strategies, we can consider that the persistence of losers is roughly more emphasised.

Lastly, let's assess the hedge funds' annual performance persistence. The favourable position of the Event Driven index seems stable over the study period. As for the arbitrage index, it shows a persistence of its loser position during 3 consecutive years. However, in cases of the global hedge fund and Directional/Tactical indexes, the annual performance persistence is rejected at the 5% significance level with Sortino and Extreme Sharpe ratios.

Table 4. The Kolmogorov-Smirnov's statistics of different hedge fund S&P indexes given different performance measures and time horizons (quarterly, half-yearly, annual)

		Sharpe ratio		Sortino Ratio		Modified Sharpe ratio 95%		Extreme Sharpe Ratio 95%		Extreme Sharpe Ratio 99%	
		Gains	Losses	Gains	Losses	Gains	Losses	Gains	Losses	Gains	Losses
three months	S&P Hedge Fund Index	**0.52** (6)	0.27 (2)	**0.44** (3)	0.19 (2)	**0.52** (6)	0.27 (2)				
	S&P Arbitrage Index	0.27 (2)	**0.57** (6)	0.27 (2)	**0.57** (6)	0.27 (2)	**0.57** (6)				
	S&P Directional/ Tactical Index	0.35 (1)	**0.77** (10)	0.27 (1)	**0.57** (6)	0.35 (1)	**0.77** (10)				
	S&P Event-Driven Index	**0.78** (7)	0.35 (1)	**0.78** (7)	0.35 (1)	**0.78** (7)	0.35 (1)				
six months	S&P Hedge Fund Index	0.42 (2)	0.25 (1)	0.42 (2)	0.25 (1)	0.42 (2)	0.25 (1)	0.42 (2)	0.25 (1)	0.13 (2)	0.13 (2)
	S&P Arbitrage Index	0.33 (1)	**0.58** (3)	0.33 (1)	**0.58** (3)	0.33 (1)	**0.58** (3)	0.33 (1)	**0.58** (3)	0.33 (1)	**0.58** (3)
	S&P Directional/ Tactical Index	0.33 (1)	**0.80** (5)	0.33 (1)	**0.80** (5)	0.33 (1)	**0.80** (5)	0.33 (1)	**0.80** (5)	0.25 (1)	0.38 (3)
	S&P Event-Driven Index	**0.98** (6)	0.50 (0)	**0.98** (6)	0.50 (0)	**0.98** (6)	0.50 (0)	**0.98** (6)	0.50 (0)	**0.98** (6)	0.50 (0)
One year	S&P Hedge Fund Index	**0.88** (3)	0.50 (0)	0.42 (2)	0.25 (1)	**0.88** (3)	0.50 (0)	0.42 (1)	0.25 (1)	0.42 (1)	0.25 (1)
	S&P Arbitrage Index	0.50 (0)	**0.88** (3)	0.50 (0)	**0.88** (3)	0.50 (0)	**0.88** (3)	0.50 (0)	**0.88** (3)	0.50 (0)	**0.88** (3)
	S&P Directional/ Tactical Index	0.50 (0)	**0.88** (3)	0.25 (1)	0.42 (2)	0.50 (0)	**0.88** (3)	0.25 (1)	0.42 (1)	0.25 (1)	0.42 (1)
	S&P Event-Driven Index	**0.88** (3)	0.50 (0)	**0.88** (3)	0.50 (0)	**0.88** (3)	0.50 (0)	**0.88** (3)	0.50 (0)	**0.88** (3)	0.50 (0)

The values in brackets indicate the maximum number of successive periods where the index is winner or loser

Empirical results tend to show that Hedge Fund S&P indexes offer short term performance persistence. When the time horizon becomes longer, the persistence phenomenon tends to decrease except for Event Driven strategies. Indeed, the latter are the only ones to maintain their performance whatever the time horizon and the performance measure used. By investing in these strategies, investors can expect to get a good performance during a time horizon longer than the lockup period imposed by contracts. In such a case, performance fees are justified because the performance persistence of funds is likely to be the result of managers' skills.

Lastly, this study makes it possible to highlight the influence of the performance measure on conclusions. With the 99% Extreme Sharpe ratio for example, the performance of some indexes is persistent whereas we get a reverse conclusion with the other performance measures.

III. Conclusion

This article shows how extreme risk of hedge funds can be taken into account when their absolute performance is evaluated. By including the VaR_{EVT}, the Extreme Sharpe ratio enables to penalise portfolios with heavy losses. This performance indicator is therefore useful to extreme risk-averse investors. It appears more relevant than Sharpe, Sortino and Modified Sharpe ratios which tend to overestimate performance of some alternative strategies. The risk measures of these usual performance indicators aren't accurate to assess extreme risk of hedge funds. To avoid over-evaluating risky portfolios such as hedge funds, investors should therefore use the Extreme Sharpe ratio which includes a right extreme risk estimator (Fromont [2007]). Nevertheless, results show that performance rankings obtained from Extreme Sharpe ratio depend on the minimum acceptable return and the VaR_{EVT} confidence level. Investors with an extreme risk aversion must therefore attach importance to the choice of these parameters. This research emphasises the original perspectives offered by the Extreme Value Theory to solve problems in the performance evaluation of portfolios having a leptokurtic and asymmetric return distribution.

Moreover, empirical evidence leads to wonder about the hedge fund capacity to have a persistent performance. It seems the performance of hedge fund S&P indexes can't manage to persist in the long term. Their performance persistence appears a short term phenomenon which tends to challenge the idea that every hedge fund managers can create value whatever the market conditions. This research also highlights the importance of studying the performance persistence of portfolios before making any investment decisions. The performance persistence of selected funds must be at least equal to the lockup period imposed in contract.

At last, let's keep in mind that results of this study would deserve to be generalised by analysing performance persistence of individual funds. Moreover, it would be interesting in future researches to carry out new applications with different time periods and various market conditions such as the financial crisis of 2008 which constitutes a life-size test. ■

Bibliography

- Amenc N., P. Malaise, L. Martellini and M. Vaissei [2004], Fund of hedge fund Reporting, Discussion Paper Edhec publication.
- Agarwal V. and N.Y. Naik [2000b], Multi-Period Performance Persistence Analysis of Hedge Funds, *Journal of Financial and Quantitative Analysis*, vol.35, pp. 327-342.
- Agarwal V. and N.Y. Naik [2000c], On Taking the Alternative Route: Risks, Rewards, and Performance Persistence of Hedge Funds, *Journal of Alternative Investments*, vol. 2, pp. 6-23.
- Amin G.S, and H.M. Kat [2001], Hedge Funds Attrition and Survivorship Bias aver the period 1994-2001, Working Paper, Swiss Federal Institute of Technology
- Backmann J.F. and S. Scholz [2003], Alternative Performance Measures of Hedge Funds, *AIMA Journal*, June.
- Brorsen B.W. and A. Harri [2004], Performance Persistence and the source of returns for hedge funds, *Applied Financial Economics*, vol. 14, pp. 131-141
- Brown S., W. Goetzmann and R. Ibbotson [1999], Offshore Hedge Funds: Survival and Performance: 1989-95, *The Journal of Business*, vol. 72, January, pp. 91-117
- Caglayan M., and F. Edwards [2001], Hedge Fund Performance and Manager Skill, *Journal of Futures Markets*, vol. 21, pp. 1003-102
- Degroot M.H. and M. J. Schervish [2002], Probability and Statistics, *Addison-Wesley*
- De Souza C. and S. Gokcan [2004], Hedge Fund Investing: A Quantitative Approach to Hedge Fund Manager Selection and De-selection, *The Journal of Wealth Management*.
- Edhec [2004], European Alternative Multimanagement Practices Survey, Edhec Publication, February
- Fishburn P.C. [1977], Mean-Risk Analysis with Risk Associated with Below Market Returns, *American Economic Review*, 67, pp. 116-126.
- Fromont E. [2005], Modélisation des rentabilités extrêmes des distributions de hedge funds, Banque & Marchés, numéro 79, pp. 31-41.
- Fromont E. [2007], La VaR_{EVT}: une mesure fiable du risque extrême de hedge funds, *Banque & Marchés*, numéro 89, pp. pp.28-36.
- Gregoriou G. and J.P. Gueyie [2003], Risk-Adjusted Performance of Funds of Hedge Funds Using a Modified Sharpe Ratio, *Journal of Wealth Management*, vol. 6, pp. 77-84.
- Kahneman D. and A. Tversky [1979], Prospect theory: An analysis of decision under risk, *Econometrica*, vol. 47, pp. 276-287.
- Kat H.M [2003], The Dangers of Mechanical Investment Decision-Making: The Case of Hedge Funds, Working Paper, City University London, November.
- Kat H.M. and F. Menexe [2003], Performance Evaluation and Conditioning Information: The Case of hedge funds, Working Paper, December.
- Keating C. and W. Shadwick [2002], A Universal Performance Measure, *Journal of Performance Measurement*, Spring, Vol.6, pp. 59-84.
- Ko F., T.H. Ko. and M. Teo [2003], Asian Hedge Funds: Return Persistence, Style and Fund Characteristics, Working Paper, June.
- Kouwenberg R. [2003], Do hedge funds Add Value to a Passive Portfolio?, *The journal of Asset Management*, vol. 3, pp. 361-382.
- Liang B. [2000], Hedge Funds: The Living and the Dead, *Journal of Financial and Quantitative Analysis*, vol. 35, N° 3, pp. 309-326.
- Mitchell M. and T. Pulvino [2001], Characteristics of Risk and Return in Risk Arbitrage, *Journal of Finance*, vol. 56, pp. 2135-2175.
- Park J.M. and J.C. Staum [1998], Performance Persistence in the Alternative Investment Industry, Working Paper, December.
- Pickands J. [1975], Statistical inference using extreme order statistics, *Annals of Statistics*, vol. 3, pp. 119-131
- Pratt J. and R. Zeckhauser [1987], Proper risk aversion, *Econometrica*, vol. 55 (1), pp. 143-154
- Scott R. and P. Horvath [1980], On the direction of reference for moments of higher order than the variance, Journal of Finance, vol. 35(4), pp. 915-919.
- Sharpe W.F [1966], Mutual Fund Performance, *Journal of Business*, January, pp. 119-138.
- Sortino F., R. van der Meer and A. Plantinga [1999], The Dutch Triangle: A Framework to Measure Upside Potential Relative to Downside Risk, *Journal of Portfolio Management*, vol. 26, pp. 50-58.