On the optimality of averaging strategies for a loss averse investor

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I Introduction

The standard way to analyze the aggregate stock market is the consumption-based approach. Agents are characterized by utility functions over consumption and they are assumed to live an infinite number of periods. In this framework, it is difficult to explain the long-term behavior of asset prices, especially the size of the equity premium. It has been the subject of intensive research since the seminal paper by Mehra and Prescott (1985). Many contributions have tried to refine agent's preferences, for example by introducing recursive utility functions [1] or habit formation [2].

These alternative models are still based on the usual assumption that utility is derived from consumption only.

More recently, an alternative approach has developed, which considers that, if utility is derived from consumption, it is also generated by variations in wealth (Barberis *et al.*, 2001). The theoretical basis for this characterization of preferences is cumulative prospect theory (Kahneman-Tversky (1979) and Tversky-Kahneman (1992)). One of the essential features of prospect theory is that gains and losses generate asymmetric effects on the agent's welfare. More precisely, agents are more sensitive to reductions in wealth than to improvements in their financial situation. This feature is called loss aversion. Moreover, the house money effect may be non negligible, that is to say, loss aversion may vary through time, depending on the preceding performance of the agent's portfolio. After gains, an investor becomes less loss averse and future potential losses are perceived as more easily bearable.

Using this description of preferences, Barberis *et al.* (2001) show that the equity premium puzzle may be explained with reasonable parameter values. Benartzi and Thaler (1995), a few years before, remarked that, even if an agent has a long horizon, he values his portfolio at regular intervals and these intermediate valuations generate utility (or disutility) due to changes in the wealth level. They deduce that the high equity premium can be explained by this myopic loss aversion. Simulations based on S&P500 returns lead these authors to conclude that an evaluation period around one year justifies the 6% equity premium over the period 1889-1978, and even more over the last century (Kocherlakota, 1996).

In this paper, we are interested in a different but related problem. When agents decide to save, they can choose among different asset categories but they can also select different rythms of saving. In fact, four popular strategies are usually considered ; lump-sum investment (referred to as LS in the following), Buy and Hold strategy (BH), Value Averaging (VA) and, finally, Dollar Cost Averaging (DCA). Strategy LS requires to invest all the initial wealth in the risky portfolio at date 0; when the BH strategy is implemented, the investor shares the initial wealth into two parts, one being invested in the risky portfolio, the other in the risk-free asset. It is a static strategy in that the investor doesn't adjust the portfolio through time. VA and DCA are two strategies leading to invest an increasing proportion of wealth in the risky portfolio as time passes. In a DCA strategy implemented over one year on a monthly basis, the investor puts one twelfth of his initial wealth in the risky portfolio each month, the remaining being invested in the risk-free asset. For VA, the idea is the same, except that a given amount W, to be invested in the risky portfolio at the end of the twelve months, is initially defined, Then, each month, the investor manages the portfolio such that $nW/12$ is invested in the risky portfolio at the end of month n. All these strategies are defined precisely in section 2.

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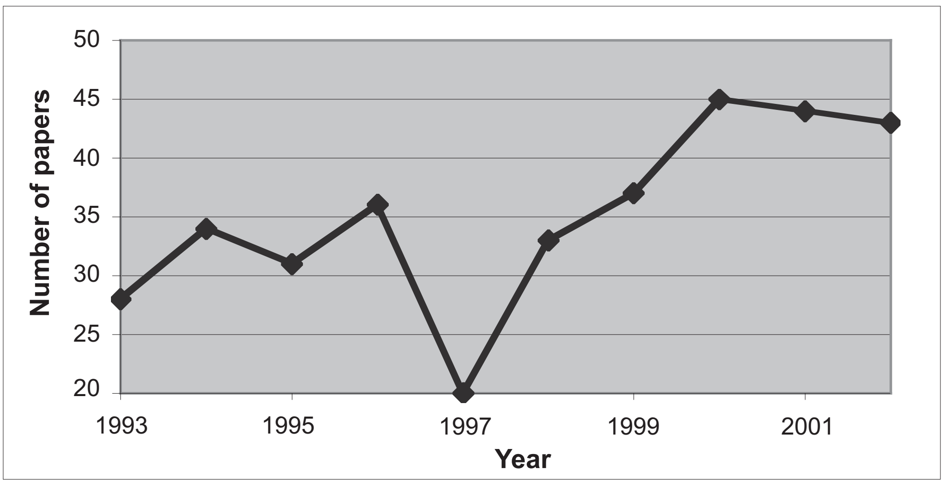
Constantinides (1979) was the first to prove that DCA is not optimal for a mean-variance investor. This first theoretical work has been followed by several empirical ones leading to the same results [3], that is to say, DCA is dominated by LS or BH. Milevsky and Posner (1999) have shown that DCA terminal payments can be duplicated by an Asian option on the mean with a zero strike price. They exhibit a BH strategy that gives the same expected return and a lower variance than DCA, provided that the return volatility is not too high. Merton (1969) and Samuelson (1969) also proved that the BH strategy is optimal for an investor endowed with a CRRA utility function, when stock prices are governed by a geometric Brownian motion.

Despite all these results, DCA remains popular, especially in bear markets. For example, *figure 1* reports the evolution of the number of papers containing the expression "dollar-cost averaging" on the ABI Inform database. It is easily seen that this number stays around 35 on the period 1993-1999 (except 1997) and jumps to 45 in 2000, remaining at high levels in the two subsequent years. Obviously, it may be a coincidence or due to a sudden jump in the number of publications referenced by ABI Inform. However, this evolution is intuitively appealing. Sellers of financial products based on DCA have essentially two arguments; the first one is that the average cost of stocks is lower than the average price [4] and the second is the reduction of variance in terminal wealth, compared to a lump-sum investment. It is then easily understandable that investors having experienced large drops in their portfolio value become more attracted by such products.

It remains that a fully rational investor in the sense of the usual expected utility model doesn't choose DCA because it is a dominated strategy. Concluding that all these investors are irrational is perhaps too easy an answer. Consequently, in this paper we analyze the four abovementioned strategies in the light of prospect theory. This approach has been already adopted by Leggio and Lien (2001) in an empirical study based, first, on thirty years of S&P500 returns (1970-1999) and, second, on the period 1926-1999. Their conclusion is that loss aversion is not able to explain the common use of DCA; in other words, DCA is not optimal for loss averse investors during the period under study. However, they considered simplifying assumptions such as a constant risk-free rate equal to the mean of risk-free rates over the periods.

Figure 1. **Papers containing "Dollar-Cost Averaging"**



Moreover, their approach of prospect theory didn't take into account the transformations of cumulative and decumulative distribution functions included in the second version of prospect theory (Tversky and Kahneman, 1992). This effect, combined with loss aversion, may be a source of explanation for the attractiveness of DCA strategies despite their sub-optimality, in the usual framework of expected utility models.

For the sake of comparability, we will use the same data as Leggio and Lien except for two points. First, our period of study ends in December 2001 and, second, our risk-free rate is in fact only locally risk-free. The Sharpe ratios for the LS and BH strategies are then different in our empirical results ; it is not the case in Leggio and Lien's paper.

We also introduce market imperfections and, more precisely, different borrowing and lending rates. It is justified if DCA is considered as the reference strategy. In other words, if it is assumed that agents save a given amount each month, they have to borrow on the market if they want to enter a lump-sum investment or a buy-and-hold strategy. We then show that a reasonable spread between borrowing and lending rates make agents almost indifferent between the alternative strategies under study.

The paper is organized as follows. The next section develops the methodology. We briefly summarize the characteristics of the four investment strategies under study and present the essential features of prospect theory to be used in our empirical analysis. The results appear in section 3. The last section concludes the paper and gives insights for future research.

II Methodology

2.1 The investment strategies

To briefly describe the four investment strategies, we consider an agent possessing an initial wealth W_0 to be shared between a risky portfolio, say a market index, and a locally risk-free asset. Depending on the strategy to be used, the investor reallocates his portfolio at each date up to his horizon and finally evaluates his return. We will use the following notations :

– T is the number of periods (years in general) under study
– n is the number of sub-periods in each period (n = 12 months in the usual case).
– r_{ti} is the risk-free return in period t and sub-period i
– R_{ti} is the corresponding return on the risky portfolio.
– n and R_t denote the compounded returns per period.
– W_{ti} is the end of period wealth (period t, sub-period i). When $i = 0$, W_{t0} is the initial wealth.

2.1.1 The lump sum investment

It consists simply in investing W_{t0} at date 0 in the market index. The return is then the index return over a year. It is calculated as usual:

$$R_t = \prod_{i=1}^{n} (1 + R_{ti}) - 1$$

The most important inconvenient of such an all-or-nothing strategy is that the portfolio can be bought at a market high, the investor bearing the losses due to a possible future drop. As long as stocks exhibit a positive premium, LS is expected to generate the highest risk-return pair on a sufficiently long holding period.

2.1.2 Buy and hold strategy

In this case, as in the preceding one, there is no dynamic management of the portfolio but the investor shares his initial wealth between the two available portfolios. For example, he chooses to invest 60% in the market index and 40% in the locally risk-free asset; at the end of the period, he computes his return and adjusts it for risk. However, even without intermediate trading, the risk of such a portfolio evolves through time, due to the dynamics of the relative values of the two parts. For example, assume that $W_0 = 1000$ is shared in $W_0^r = 600$ and $W_0^f = 400$ (corresponding to the above percentages). After six months (at time $t = 0.5$ on a yearly basis), assume that the return on the risk-free asset is 2% when the return on the risky asset is 6%. The total wealth is then:

$$W_{\frac{1}{2}} = 600 \times 1.06 + 400 \times 1.02 = 1044$$

and the proportions invested in the two assets are now $\frac{636}{1044} = 60.92\%$ in the risky asset and $\frac{408}{1044} = 39.08\%$ in the risk-free asset.

The proportion of wealth invested in the risky asset increases when the stock return is greater than the risk-free rate; it is the case on real long-term data. As Milevski and Posner (1999) found that a BH strategy with a proportion $\frac{1}{2} - \left(\frac{\mu - r}{12}\right)$ invested in the risky asset (with μ the expected return on the stock and r the risk-free rate) dominates the DCA portfolio in a mean-variance world, we impose $x = 50\%$ to be the proportion invested in the risky asset. In fact, on the basis of historical returns, the equity premium is around 6%; with such a figure, the proportion of Milevsky and Posner is equal to 49.5%.

Table 1 gives an illustration of the evolution of a BH portfolio during the year 2001 when the initial wealth is 100 and the proportion of initial wealth invested in the risky portfolio (the S&P500) is 50%.

Table 1. **Evolution of a BH portfolio**

Month	Index Return (in %)	Risk-free Return (in %)	BH portfolio value
1	3.55	0.54	102.045
2	-9.12	0.38	97.514
3	-6.34	0.42	94.743
4	7.77	0.39	98.365
5	0.67	0.32	98.846
6	-2.43	0.28	97.827
7	-0.98	0.3	97.523
8	-6.26	0.31	94.791
9	-8.08	0.28	91.436
10	1.91	0.22	92.310
11	7.67	0.17	95.509
12	0.88	0.15	95.971

2.1.3 The value averaging strategy

The intuitive idea behind value averaging is to buy at low prices (stated in this way, it is surely a good strategy!). In fact, the investor chooses to smooth the evolution of the value of his risky portfolio through time.

To do this, he defines a desired terminal value W_{tn} to be invested in the risky asset. He starts by investing $\frac{W_{tn}}{n}$ in the risky portfolio and $W_{t0} - \frac{W_{tn}}{n}$ in the risk-free asset at the beginning of the first sub-period. At date 1, let W_{t1} be value of the risky portfolio; the second period investment in the risky asset is then $2\frac{W_{tn}}{n} - W_{t1}$. The process continues up to the investment horizon.

To illustrate the idea governing this type of strategy, assume that the monthly risk-free rate is 0.25% and the initial wealth is $W_{t0} = 10\,000$ with a terminal desired level $W_{tn} = 10\,800$. *Table 2* gives a possible evolution for the portfolio value. This simulation is realized with a gaussian risky return with a yearly mean equal to 9% and a corresponding standard deviation equal to 20%.

It appears that the investment in the risky portfolio is greater after market drops so the investor buys more stocks at “low” prices. As the strategy is self-financed, the net investment in the risky asset is realized by transferring the necessary amount from the risk-free account. For example, at the end of the first period, we have $W_{t1}^r = 927.92$ and

Table 2. **Simulation of a value averaged portfolio**

Date	Portfolio value	Market index	Risk-free asset	Risk-free return	Market return
0	10000	900	9100	0.25%	3.05%
1	10050.7	1800	8250.7	0.25%	-3.93%
2	10001.92	2700	7301.92	0.25%	13%
3	10394.94	3600	6794.94	0.25%	4.41%
4	10574.2	4500	6074.2	0.25%	7.56%
5	10942.88	5400	5542.88	0.25%	-5.28%
6	10679.01	6300	4379.01	0.25%	11.56%
7	11461.74	7200	4261.74	0.25%	-0.3%
8	11450.62	8100	3350.62	0.25%	-8.22%
9	10820.02	9000	1820.02	0.25%	-1.63%
10	10679.32	9900	779.32	0.25%	-11.58%
11	9598.57	10800	-1201.43	0.25%	-3.78%

$W_{t1}^f = 9\ 122.78$ where superscripts *r* and *f* stand for risky and risk-free. Consequently, the agent takes away 1 800 – 927.92 = 872.08 from the riskless account (which then evolves to 9 122.78 – 872.08 = 8 250.7) to invest in the risky asset. It is worth noticing that the final wealth invested in the risk-free asset may be negative if the risky asset performs poorly at intermediate dates. It means that an implicit assumption is equal borrowing and lending rates.

VA is a dynamic strategy in the spirit of CPPI (Constant Proportion Portfolio Insurance) introduced by Perold (1986) and Perold and Sharpe (1988). However, in the CPPI approach, the investor defines a floor for the value of the portfolio. In VA, he initially defines an amount to be invested in the risky asset at the end of the period under consideration.

2.1.4 Dollar-cost averaging

This strategy simply consists in investing the same amount in the risky asset in each sub-period. More precisely, assume an investor starts the year with $W_{t0} = \$12\ 000$. He first invests $\frac{W_{t0}}{12} = \$1\ 000$ in the risky asset and $\frac{11}{12}W_{t0} = \$11\ 000$ in the risk-free asset. He then obtains

$$W_{t1} = \frac{W_{t0}}{12}(1+R_{t1}) + \frac{11W_{t0}}{12}(1+r_{t1})$$

at the end of the first period. He then invest $W_{t1} - \frac{10W_{t0}}{12}$ in the risky asset and so on, up to the end of the year. Consequently, at the beginning of the last sub-period, all the initial wealth is invested in the risky asset. Hence, as in the VA case, the number of stocks bought is greater when prices are low. It must be noticed that the new amount invested in the risky asset is not exactly equal in each period, due to the interests earned on the risk-free account.

As mentioned before, the usual arguments in favour of DCA are the risk reduction it generates, when compared to the LS strategy and also, a more questionable argument, the mean buying cost which is lower than the average price of the stock. In fact, consider an investor endowed with \$1 000, a risky asset which is worth $S_0 = \$100$ at date 0 and the risk-free rate is 0; assume further that a two-period DCA strategy is implemented. 5 units of stock are bought at date 0. If at date 1, $S_1 = 50$, ten more stocks are purchased. The mean buying cost is then 1000/15 = \$66.66 when the mean price is \$75. The same argument applies when the stock price increases. For example, if the stock price jumps to \$200, the final portfolio contains 7.5 stocks and the mean unit cost is \$133.33 with a mean price equal to \$150. More generally, this property comes from the general relationship between the harmonic mean and the arithmetic mean, the former being always lower than the latter.

Another important argument, not often referred to in the theoretical literature, is that the investor doesn't need W_0 at the beginning of the first sub-period. Only $\frac{W_0}{n}$ is in fact needed at date 0 to enter DCA. If W_0 is the amount to be invested (the reference amount), then $\frac{n-1}{n}W_0$ must be borrowed to implement the strategy. In section 3.4, we will show that if the reference wealth is a sequence of *n* successive amounts of savings $\frac{W_0}{n}$, borrowing is necessary to implement the LS or BH strategies, leading to highly leveraged (and then risky) positions in these cases.

2.2 *The performance measures*

2.2.1 The Sharpe ratio

The usual Sharpe ratio in period *t* is denoted by s_t and measured as :

$$s_t = \frac{\frac{W_{tn}}{W_{t0}} - \prod_{i=1}^{n}(1+r_{ti})}{\sigma_t}$$

where σ_t is the estimated standard deviation of period-*t* returns. If we define $\rho_{ti} = \frac{W_{ti}}{W_{ti-1}}$, we get :

$$\sigma_t^2 = \frac{1}{n-1}\sum_{i=1}^{n}(\rho_{ti} - \bar{\rho}_t)^2$$

with $\bar{\rho}_t = \frac{1}{n}\sum_{i=1}^{n}\rho_{ti}$.

It must be noted that, as we will use the S&P500 index as the risky asset and the Treasury bills return as the locally risk-free asset, the Sharpe rations of LS and BH will be different, contrary to the assumption of Leggio and Lien (2001) who used an annualized risk-free rate of return and then get identical Sharpe ratios for the two strategies.

2.2.2 The valuation function of prospect theory

The four essential features of prospect theory are the following:

1) Investors value risky prospects with respect to a reference point which can be current wealth, when the outcomes are obtained immediately, or current wealth capitalized at the risk-free rate when outcomes are obtained at the end of the period under consideration.
2) Losses and gains are weighted differently in the valuation function. The perceived disutility of a $100 loss is greater than the utility of $100 gain.
3) Agents are risk-averse in the domain of gains and risk-lovers in the domain of losses. In other words, when a lottery generates losses, agents are ready to gamble to avoid the most extreme losses.
4) Investors tend to overweigh small probability events.

These four characteristics were pointed out by many experiments, reported in Kahneman-Tversky (1979) and Tversky-Kahneman (1992).

It is worth noticing that the question of the reference point leads to difficult theoretical questions since it may appear preference reversals when the reference point is changed.

These elements are formalized in the following way. Consider a lottery $X = ((x_i, p_i), i = 1\ldots, n)$ such that $x_1 < \ldots < x_m = 0 < x_{m+1} < \ldots < x_n$.

The valuation function V is defined as :

$$V(X) = V(X^+) + V(X^-)$$

The valuation function is shared into two parts, to take into account the asymmetry between gains and losses. $V(X^+)$ and $V(X^-)$ are defined as follows :

$$V(X^+) = \sum_{i=m}^{n} \pi_i^+ v(x_i) \text{ and } V(X^-) = \sum_{i=1}^{m} \pi_i^- v(x_i)$$

The decision weights π^+ and π^- are defined by means of two weighting functions w^+ and w^- :

$$\pi_n^+ = w^+(p_n) \ ; \ \pi_i^+ = w^+\left(\sum_{j=i}^{n} p_j\right) - w^+\left(\sum_{j=i+1}^{n} p_j\right)$$

$$\text{for } m \le i < n$$

$$\pi_n^- = w^-(p_1) \ ; \ \pi_n^- = w^-\left(\sum_{j=i}^{i} p_j\right) - w^-\left(\sum_{j=1}^{i-1} p_j\right)$$

$$\text{for } 0 \le i \le m$$

It is worth noticing that w^+ is applied to the decumulative distribution function when w^- is applied to the cumulative distribution function.

The parametric form used by Tversky and Kahneman is the same for the two functions :

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$$

However, the parameter γ takes two different values; based on experimental data, Tversky and Kahneman obtain $\gamma^+ = 0.61$ and $\gamma^- = 0.69$.

The function w was first proposed by Quiggin (1982) in the Rank Dependent Expected Utility model to take into account the transformation of probabilities referred to in point (4). *Figure 2* illustrates the shape of w for $\gamma = 0.61$. As w is applied to cumulative or decumulative distribution functions, it appears that small probability events are over-weighted and large probability events are under-weighted by w.

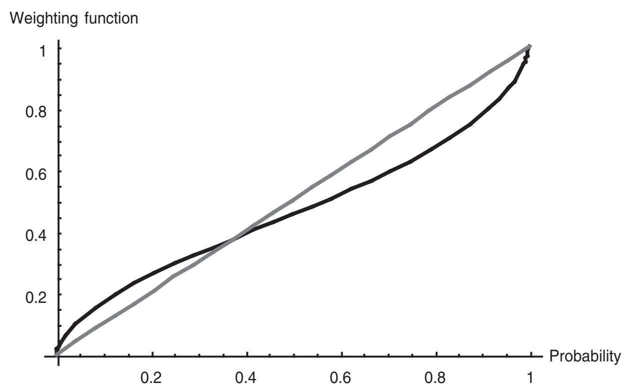
Concerning the value function v, the usual parametric form is :

$$v(x) = \begin{cases} (x - x^*)^\alpha & if \quad x \ge x^* \\ -\lambda(x^* - x)^\alpha & if \quad x < x^* \end{cases}$$

where x^* is the reference point. The original estimation of the parameters was $\alpha = \beta = 0.88$ and $\lambda = 2.25$. A coefficient $\lambda > 1$ is aimed at taking into account loss aversion. Moreover, α and β, lower than one, imply risk aversion over gains and risk loving over losses. To sum up, the valuation function V takes into account the abovementioned features of observed behavior.

In our study, the standard period-length is one year, divided into twelve months. Consequently, if W_{t0} is the initial wealth in period t, $W_{t0}\prod_{i=1}^{n}(1 + r_{ti})$ is the reference level, calculated as the final wealth, obtained after a rolling investment in the locally risk-free asset.

Figure 2. **An example of weighting function**



There are two ways to perform the analysis of historical data by means of the above valuation function. The first is to compute the valuation function *ex post* for each year and then evaluate the mean. It is essentially the methodology used by Leggio and Lien (2001). For the sake of comparison, we will present the results obtained in this approach. However, the spirit of cumulative prospect theory implies to base results on $V(X)$, that is on an *ex ante* distribution of returns. The same remark obviously applies to any theory of portfolio management. The question is then how to estimate the distribution, relying on historical data. In their study of the equity premium puzzle, Bernartzi and Thaler (1995) used simulations by drawing samples of monthly returns with replacement. The implicit assumption is that successive returns are independent random variables. An alternative method, used in the following, is to estimate the distribution of returns by considering all the possible successive 12-month periods in the data; the returns obtained for each of the four strategies are then ranked and equally weighted. The functions π^+ and π^- are applied to these sequences. The problem Benartzi and Thaler were studying was the determination of the evaluation period, linked to myopic loss aversion. In other words, the equity premium may be explained by the fact that investors evaluate their portfolios at intermediate dates and get "utility" from these valuations. These authors, studying the period 1926-1990 found an evaluation period of around 12 months. Our approach is consistent with this finding; in fact, for twelve-month periods, we find $V(X) = -0.002$ and for thirteen-month periods, $V(X) = 0.0033$, meaning that the evaluation period is between the two. The sample for this computation is the S&P500 and the strategy considered is the LS one for comparison puposes.

III Empirical results

3.1 The data

The data used in this study are total returns on the S&P500 large company stocks and the Ibbotson small company stocks, provided by the 2002 Valuation Edition of "Stocks, Bonds, Bills and Inflation" (Ibbotson Associates). The risk-free rate is the US Treasury bills total return coming from the same source. The period under consideration is 1926-2001, that is 76 years. The results we present hereafter are decomposed into two categories. Fist, we analyse the 30-year period (1970-1999) for the two kinds of companies, to get some elements of comparison with Leggio and Lien (2001). We then present the results for the overall period of 76 years. The following tables report the means of the Sharpe ratio, the KT valuation function, the compounded return and the standard deviation of this return for each strategy. However, we only report here the results for large stocks ; from the qualitative point of view, they are similar to the ones obtained for small stocks [5].

For example, when the overall period is analysed, the performance of the four strategies is evaluated on the $76 \times 12 - 11 = 901$ successive 12-month periods. On a statistical point of view, it means that the characteristic parameters presented in the tables do not come from an independent sample but it is not our main concern in this paper. Doing so is aimed at avoiding some disturbing effects such as the January effect. It may be important for the two dynamic strategies because the investment in the risky asset is increased each month up to the end of the year. As these results give the mean value of the KT function, evaluated at the end of each year, the weighting function is not taken into account in this part.

The second category of results introduces weights. To manage this task, it is necessary to evaluate an *ex ante* distribution of returns for the different strategies. We then consider an investor facing a one-year investment in one of the four strategies. To simulate the distribution of returns, we consider the 901 yearly returns obtained from the historical data and consider that it is the set of possible returns for the year to come. In other words, it is as if the investor was choosing a strategy without knowing the point in time he is. The alternative choice would be to pick up a sample of series of 12 monthly returns among the 912 available returns. The essential advantage of our approach is to take into account the possible short-term serial correlations of returns. For example, there are 910 subsets of three successive months and we would expect that 25% (under the independence assumption) of them contain three returns of the same sign. In fact, we get 31,31% of such series.

3.2 Results for the 1970-1999 period

Table 3 presents the first results for large stocks. Even if, from a quantitative point of view, the results are different from those of Leggio and Lien (2001), due to distinct methodological assumptions, our results also show the superiority of the lump sum strategy over the three others; it may seem surprising because LS generates a greater range of returns and, consequently, greater losses for some years, (losses are weighted by $\lambda = 2.25$ in the KT valuation function). In fact, the penalty for losses is largely compensated by the mean excess return which is around 8.5%. It is about two times the excess return for the other strategies.

Table 3. **Large stocks : 1970-1999**

Large stocks	LS	BH	VA	DCA
Sharpe ration	0.727	0.682	0.695	0.660
KT function	2.917	1.585	1.369	1.341
Std deviation	14.38%	7.21%	8.22%	8.77%
Mean return	8.49%	4.24%	4.39%	4.71%
% of positive excess returns	72.5	72.5	72.5	71.1
KT weighted function	-0.0022	0.0012	-0.0176	-0.0135
Highest return	52.47%	26.24%	24.75%	32%
Lowest return	-47.03%	-23.52%	-38.12%	-34.63%

Concerning the sign of the excess returns, it appears no significant differences in the proportion of positive excess returns across strategies, a not surprising result when taking into account the fact that the two dynamic strategies (VA and DCA) lead to an increase in the proportion of stocks in the portfolio as time passes.

The following line in the table gives the value of the KT weighted function, as described in section 2.2.2. Function v is applied to the distribution of returns. It explains the difference in magnitude with respect to the KT function which was calculated on an initial wealth of 100. In other words, the reported figures are the expected weighted KT value function for an initial investment equal to 1. Here, the results are clear-cut. Based on historical returns, an agent obeying prospect theory will not enter an "average" strategy because his expected value function is negative. More over, BH is the only strategy an investor would use since LS has also a negative value function.

3.3 *Results for the 1926-2001 period*

Now the period under study contains 912 months, that is 901 periods of twelve months. The results are presented in *table 4*. There are some important differences on this longer period. First, Sharpe ratios are greater, even if the proportion of periods with positive excess returns is comparable to the one obtained in the 1970-1999 period. The most important difference is that all the weighted functions take on negative values. This result is due to the introduction of periods with very large drops, for example the years following the 1929 stock market krach.

The effect of loss aversion then appears clearly but it doesn't change the ranking of strategies. In the light of such results and those already published in the literature, it is hard to explain the popularity of averaging strategies. Even when taking into account the transformation of probabilities and the asymmetry between losses and gains, LS and BH remain superior.

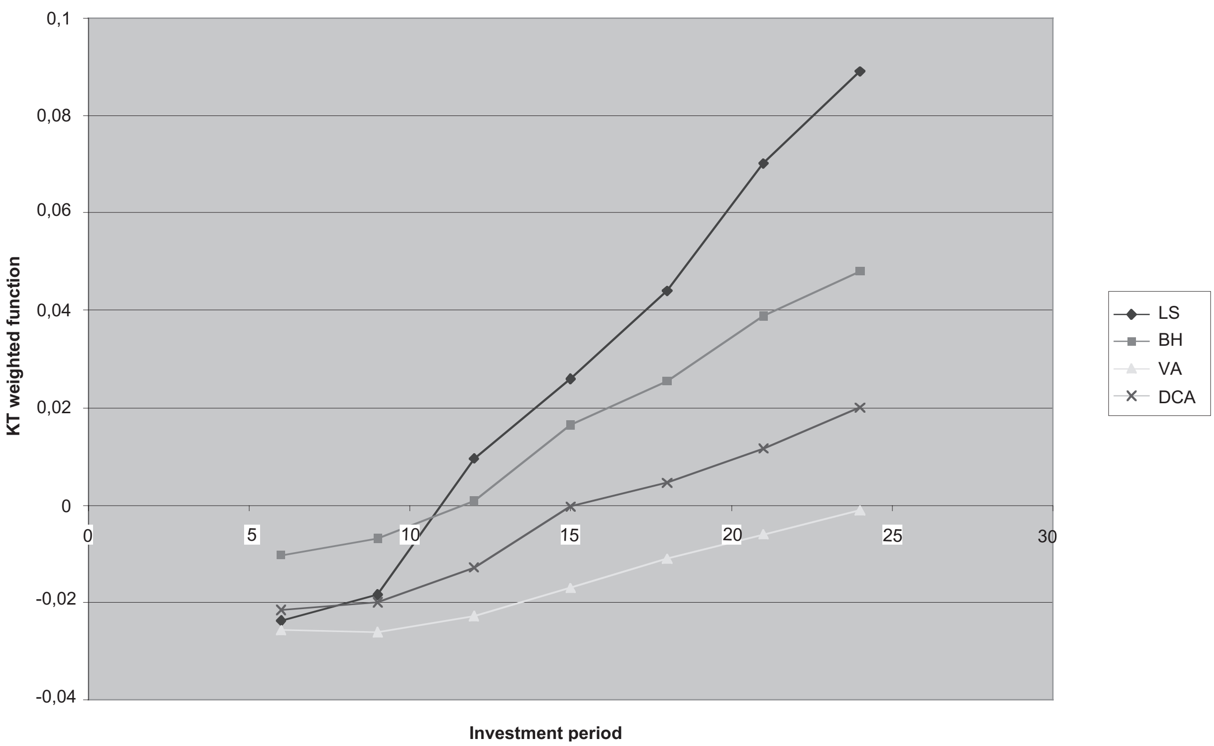
Several natural questions remain at that stage. The first one concerns the estimation of the parameters. We kept the original estimations but it may be that loss aversion is greater than 2.25. However, applying the same method with $\lambda = 3$ leads to the same results in ordinal terms. Obviously, the KT functions decrease but the static strategies are still better. It can be observed that the buy and hold strategy becomes the best when λ increases, a not too surprising result.

Table 4. **Large stocks : 1926-2001**

Large stocks	LS	BH	VA	DCA
Sharpe ration	0.859	0.795	0.772	0.729
KT function	2.119	1.151	0.763	0.768
Std deviation	16,55%	8.23%	9.95%	10.13%
Mean return	9.26%	4.63%	4.6%	4.89%
% of positive excess returns	68.5	68.5	70.1	68.3
KT weighted function	-0.0008	-0.0004	-0.0327	-0.0237
Highest return	162.56%	81.28%	60.8%	68.41%
Lowest return	-68.89%	-34.45%	-66.40%	-51.80%

The second question is the arbitrary choice of one year to implement these strategies. In fact, other choices do not alter the qualitative nature of the results. *Figure 3* shows the evolution of the KT function for the four strategies when the reference period varies from 6 to 24 months by 3-month steps. It is quite clear that BH always dominate DCA and VA, confirming one more time the result obtained in the expected utility framework.

Figure 3. **KT functions versus investment periods**



Finally, we can question the choice of the period. Is the ranking of strategies the same on bull and bear periods. To illustrate this point we arbitrarily selected two periods. The first one is 1926-1932; a dollar invested in the S&P500 in January 1926 was worth 0.788 in December 1932. The second period is 1990-1999. A dollar invested in the same index at the beginning of January 1990 was worth 5.323 at the end of 1999. *Table 5* shows the values of the weighted KT functions for the two subperiods. It clearly appears that the static strategies (LS and BH) dominate VA and DCA. In the bull market, not surprisingly, the LS obtains the best evaluation, the three others being very close. In the bear market, LS gets the worst performance but BH dominates VA and DCA. It appears that, at this stage, DCA cannot be justified by an alternative description of preferences.

Table 5. **Bull and bear periods**

KT weighted functions	LS	BH	VA	DCA
1926-1932	-0.274	-0.149	-0.221	-0.194
1990-1999	0.152	0.082	0.078	0.082

3.4 DCA as the reference strategy

The preceding analyses neglect an important fact in that they assume W_0 is available to the investor at the beginning of the period. It was also assumed that the four strategies were self-financed ; the unique cash-inflow was W_0 at date 0. However, for DCA to be implemented, only $\frac{W_0}{12}$ is needed at the beginning of each month. In fact, it is often presented as a strategy involving monthly savings.

An obvious answer to this remark would be to say that the perfect market assumption solves the problem, since the investor can borrow the missing funds at the risk-free rate at date 0. However, doing so introduces a leverage effect for LS and BH, which increases the standard deviation of monthly returns. For example, if agents save \$100 each month, the implementation of the LS strategy requires to borrow \$1 100 at the beginning of the first period.

In other words, the preceding sections were considering the LS strategy as the benchmark. We examine now the alternative view in which DCA is the benchmark, that is to say, we assume that agents save $\frac{W_0}{12}$ each month. Investing more than this amount in the risky asset implies to borrow at the risk-free rate. It will obviously change the Sharpe ratio and the KT function as well. Hereafter, we focus the analysis on LS, BH and DCA and we introduce a spread between borrowing and lending rates, denoted as *s*.

To give an illustration of what happens in this case, consider the terminal wealth at the end of the first month for the three strategies in year *t*, divided into *n* subperiods (*n* = 12 if the subperiods are months). They are given by :

$$W_{t1}^{LS} = W_0\left((1+R_{t1}) - \frac{n-1}{n}(1+r_{t1}+s)\right)$$

$$W_{t1}^{BH} = W_0\left(\frac{1}{2}(1+R_{t1}) - \frac{\frac{n}{2}-1}{n}(1+r_{t1}+s)\right)$$

$$= \frac{W_0}{2}\left((1+R_{t1}) - \frac{n-2}{n}(1+r_{t1}+s)\right)$$

$$W_{t1}^{DCA} = \frac{W_0}{n}(1+R_{t1})$$

The yearly return is calculated, as usual, as the ratio of terminal and initial wealths, minus 1. It is worth noticing that the return on DCA for the first month is the same as the LS return in the preceding section. However, in terms of wealth variations, the result is different since only $\frac{W_0}{n}$ is invested in the risky asset. For the LS strategy, the leverage effect is very important during the first month because $W_0\frac{n-1}{n}$ is borrowed at the risk-free rate, to invest W_0 in the risky portfolio. We can then expect a very high standard deviation of returns on a monthly basis. However, as the horizon is the year, the valuation function has to be evaluated once a year. The same logic applies for BH except that only $W_0\frac{n-2}{2n}$ has to be borrowed, implying a lower leverage effect.

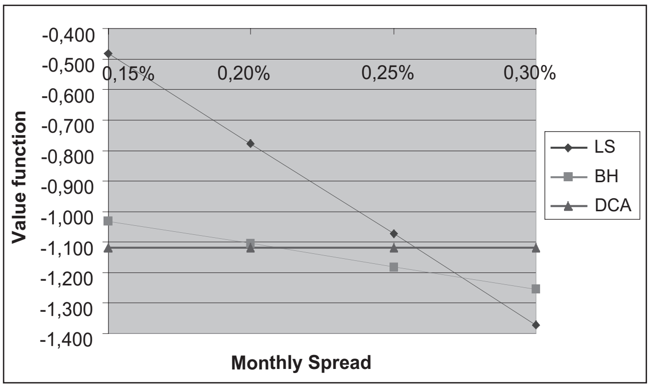
Table 6 shows the results obtained for large stocks when $s = 0$. The standard deviations are given on a monthly basis since it would be incorrect here to multiply by $\sqrt{12}$. In fact, due to the saving process, the leverage effect decreases each month as debt is progressively reimbursed. Weighted and unweighted KT functions are still ranked in the same order. It appears that one cannot explain the choice of a DCA strategy in the framework of a perfect market in which investors borrow at the risk-free rate. This result may be a surprise, considering the mean returns and the standard deviations. However, the evaluation function is computed at the end of each twelve-month period.

Table 6. **Large stocks : 1926-2001**

Large stocks 1926-2001	LS	BH	DCA
KT function	0.365	-0.818	-1.118
Std deviation (monthly)	23,08%	9.91%	4.77%
Mean return (yearly)	11.34%	6.71%	6.89%
KT weighted function	-0.025	-0.028	-0.049

The last point we want to address is the market imperfection of different borrowing and lending rates. More precisely, we will now consider that agents' rate of borrowing is greater than the risk-free rate, the difference between the two being the spread *s*. The spread penalizes LS and BH but not DCA since agents only save $\frac{W_0}{12}$ each month in the latter strategy. As returns in our database are given on a monthly basis, figure 4 give the values of the unweighted KT functions for spreads varying from 0.15% to 0.30% on a monthly basis. The value function of DCA appears as a horizontal line, due to the absence of borrowing. The two other strategies are reported as decreasing curves due to the penalization generated by the spread. The three value functions are almost equal when the spread is around 0.25%, that is to say, about 3% on a yearly basis. Obviously, using a constant spread all over the period is a rough approach since risk-free rates have varied from 2 to 18%. However, the purpose is there to show that this market imperfection allows to get a partial explanation of the popularity of DCA on a rational basis.

Figure 4. **Value functions versus borrowing spread**



IV Concluding remarks

In this study, we tried to explain why many investors enter averaging strategies, especially DCA which is a rather simple investment program. Many results in the literature showed before that DCA is not optimal for a mean-variance investor, or more generally for an investor maximizing an expected utility. In this paper, we introduced cumulative prospect theory as a descriptive model of the investor behavior because it takes into account loss aversion and distortion of probabilities, then overweighing small probability events. Two arguments are usually pointed out in favor of averaging strategies; they are based on variance reduction and "low" buying costs. However, historical data don't confirm this questionable intuition. In almost all cases, static portfolios dominate averaging strategies, especially the BH portfolio.

In the last section of the paper, we introduced market imperfections that can be faced by real investors, namely, different borrowing and lending rates. We showed that this imperfection penalizes the investors entering LS or BH. For a sufficiently high (but realistic) spread, DCA may become optimal when compared to the two static strategies.

From a theoretical point of view, the use of cumulative prospect theory in finance remains difficult because of the necessary choice of a reference point that is not initial wealth. In particular, the choice of a capitalization rate to define the reference terminal wealth is not neutral. In our analysis, choosing a long-term yield on government bonds (not presented in the paper) leads to preference reversals between VA and DCA, but not between static and dynamic strategies. Schmidt and Zank (2002) have shown that preferences are independent of the reference point when outcomes are all positive (negative) but no general result exists, to the best of our knowledge, for prospects with positive and negative outcomes. ■

1 Epstein-Zin (1989, 1991), Weil (1989).
2 Constantinides (1990), Campbell-Cochrane (1999).
3 For example Rozeff (1974), Knight and Mandell (1993).
4 See section 2.1.4 for an illustration.
5 The results for small stocks are avalable upon request.

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