The Link between Eurozone Sovereign Debt and CDS Prices

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The purpose of this paper is to examine the relationship between the prices of sovereign bonds and sovereign-linked credit default swaps (CDS) in the Eurozone. We first examine this by comparing the relative pricing of sovereign bonds and CDS at various moments in time to see if the no-arbitrage pricing relationship is obeyed. We then examine the nature of the dynamic relationship between CDS and bond markets to see if there is a lead or lag relationship, to test for cointegration and to examine for the presence of Granger causality between the two markets.

One motivation for this work has been to better understand the interrelationship between the sovereign CDS and bond market, especially in times of distress. Another is to add information for policy makers who have suggested[1], that the speculative use of CDS by market participants has caused or accelerated the rapid decline in 2010-11 of the bond prices of the Eurozone periphery countries of Portugal, Ireland, Italy, Greece and Spain, hereafter known by the acronym PIIGS. Specifically, this paper addresses two of the underlying reasons for this charge. The first is the tendency of CDS spreads to exceed bond yield spreads. For some this is seen as evidence of the belief that the CDS market is driven by the demands of protection buyers. The second is the claim that changes in the CDS spread of a specific reference entity lead the increase in the corresponding yield spread of bonds on the same reference entity.

There is a no-arbitrage relationship between the prices of credit default swap (CDS) contracts on a reference entity and the credit spreads of same currency par bonds issued by the same reference entity. In practice this relationship is held approximately and deviations may appear for either fundamental and market reasons. Fundamental reasons are those due to the exact mechanics of a bond and CDS which mean that a bond plus CDS position is not a perfect credit hedge. Market reasons relate to factors such as liquidity, and supply and demand. These factors are set out in detail in [O'Kane & McAdie 2001].

However to establish a quantitative relationship, we must introduce a credit valuation model which enables us to price bonds and CDS contracts within the same framework. This allows us to establish a direct relationship between the credit spread of bonds and CDS spreads which takes into account the contractual differences between bonds and CDS. We are then able to test whether or not a CDS spread trading wider than the corresponding bond spread signals a mispricing. One important difference from previous work is that the standard and hence the most liquid CDS contracts on Eurozone sovereign credits are denominated in US dollars. However the deliverable obligations are denominated in euros. The model must take this into account.

To examine the second question of which market leads which, we need to empirically examine the relationship between the CDS spreads and bond markets credit spreads in order to test for any lead or lag relationship. However, pure causation in a physical sense is hard to prove from time series data alone since a transmission mechanism needs to be demonstrated. Such a mechanism may or may not exist and if it does exist, it may not be easily observed. We can however test for weaker forms of causality such as Granger causality [Baltagi 2011]. This establishes whether or not there is a measure of predictive causality between two variables X and Y in the sense that past values of Y improve our ability to predict future values of X more than with just past values of X. If this is the case, then we say that Y Granger causes X. We can also test the reverse hypothesis that X Granger causes Y.

There have been a number of papers on the joint dynamics of CDS and bond spreads. A study by [Blanco et al. 2005] examined the corporate bond market. It found that CDS spreads led bond spreads and that there was a co-integrated relationship between both sets of spreads on the same reference entity. One explanation suggested by [Das 2011] is that the greater convenience of the CDS market means that price discovery occurs there before it does in the bond market. However, [Levy 2009] shows that there is little empirical support for the idea that CDS markets lead bonds in the emerging markets and that a decrease in CDS liquidity causes the CDS spread to increase while counterparty risk causes CDS spreads to decrease.

A recent paper by [Calice, Chen and Williams 2011] has examined the effect of liquidity spillovers between the CDS and government bond markets and determined that the liquidity of the CDS market has a substantial influence on sovereign debt spreads.

The structure of this paper is as follows. In the next section we set out the no-arbitrage arguments for the theoretical basis for the link between the price of CDS contracts and physical obligations on the same reference entity, and we propose a simple model to establish a link between their two different quoted spreads. We then examine reasons why this model-implied relationship can break down in practice. Following this, we perform a time series analysis of the actual CDS and bond spreads for the PIIGS and France. We look for co-integration between the CDS and bond spreads and then examine whether Granger causality is present. We then present our conclusions.

THE LINK BETWEEN BONDS AND CDS

It has been shown [Duffie 1999] that subject to some assumptions, a long position in a par priced floating rate note plus the purchase of the same face value of CDS protection, assuming this has zero initial cost, creates a combined position which has no credit risk in the event of default. These assumptions include the existence of the par floating rate note to the CDS maturity, that the funding rate of the protection buyer is Libor, that the funding can be repaid at par at the time of a default, that the delivery option has no value and we ignore the impact of the accrued bond coupon which is not protected by the CDS. It is also assumed that bond and CDS are denominated in the same currency.

Subject to these assumptions, the position is risk-free as any credit loss on the par bond is exactly matched by the payment from the CDS. Avoidance of arbitrage then implies that the (annualised) spread over Libor paid by the bond should equal the (annualised) spread paid by the protection buyer on the CDS. A link is therefore established between bond and CDS pricing. However, the strength of the link depends on the ability of market participants to implement a no-arbitrage trading strategy. As most government bonds are issued in a fixed coupon format, this no-arbitrage strategy is not usually possible. In this case, one possible strategy is to buy the fixed coupon bond as part of a par-par asset swap package [O'Kane 2008]. However, even if the bond is initially priced at par, the default risk of the asset swap cannot be hedged exactly by a CDS due to the default contingent unwind cost of the embedded interest rate swap. This means that there is no strict no-arbitrage relationship between the CDS spread and the asset swap spread. Another strategy is to buy a par valued credit risky sovereign fixed coupon, sell a par valued same maturity default-free government bond, and buy the same notional of CDS protection. The principal will be exactly hedged against a default. In this case we can think of the bond credit spread measure as the yield-to-maturity of the credit risky bond minus the yield-to-maturity of the same maturity default-free government bond.

What these hedging strategies have in common is that they only work if the bond is initially priced at par. When the price of a bond is away from par, these simple strategies are no longer hedged against principal losses. We can calculate the appropriate hedge to adjust for this effect[2]. If the bond is priced above par, the amount of protection bought needs to be scaled up to account for the larger loss in the event of a default. If the bond is priced below par then the amount of protection needs to be scaled down. However both strategies require an estimate of the future realised recovery rate of the reference credit if there is a credit event. So while there is a link between the CDS spread and the bond's yield spread, for a non-par bond, the value of the portfolio is no longer a static hedge of the bond face value since the trader is exposed if the value of the realised recovery rate following a default differs from the expected recovery used to set up the hedge.

Furthermore, since 2009, the changes in the mechanics of CDS contracts means that CDS now trade with a fixed coupon and a non-zero upfront payment. For highly distressed credits with a low fixed coupon, most of the cost of protection will be paid upfront. The price of the bond plus the cost of the upfront payment for CDS protection will mean that the strategy will usually cost more than par. The investor will therefore lose money if default occurs immediately. A trade scenario analysis will show that the trade will only make money if default occurs after some fixed horizon because sufficient time will need to pass to allow the investor to receive the higher bond coupons. This change in CDS format makes even the most standard no-arbitrage strategy even harder to achieve.

The effect of the bond trading away from par, plus the recent CDS contract changes, means that it is almost impossible to guarantee a risk-free profit from a trading strategy if a model-implied mispricing is identified, even after taking into account bid-offer spreads. Traders who do see a model-implied mispricing between CDS and bond spreads will therefore only act if the size of the mispricing and potential profit is commensurate with the risk. This means that any mispricing, even if it violates the theoretical relationship established by a model, will usually persist until it becomes large enough to look attractive on a risk-return basis.

Another contractual detail which will cause a basis between CDS and bond spreads in the context of Eurozone sovereigns is that standard traded CDS contracts on Eurozone sovereigns are denominated in US dollars while the bonds which they hedge are denominated in euros. Therefore, at initiation a euro-based protection buyer will only be able to buy a dollar denominated CDS. The dollar amount of protection purchased will depend on the initial exchange rate and the face value in euros of the exposure to be hedged. If there is a credit event; the loss compensation amount paid out on the protection leg is calculated as par minus the recovery rate on the dollar face value. The recovery rate is set by convention to be identical for both euro and US dollar denominated CDS as it is the price as a percentage of the face value of the deliverable obligations set via an auction process conducted within 2 months of the credit event. A euro-based hedger will convert the protection payment back to euros.

If a credit event on a Eurozone sovereign does occur, we might expect that there will be a shock which could cause the Eurodollar exchange rate to change. If the market believes that the euro will weaken against the US dollar, this will result in a windfall profit for the euro-based hedger. To prevent investors from systematically buying protection in dollars and making money from a credit event which leads to a devaluation, the dollar-denominated spread must embed the market view on the expected change in the exchange rate *conditional on* a credit event. If the market sees devaluation as being likely then this will cause dollar denominated CDS spreads to trade at higher levels than the (non-standard and less liquid) euro-denominated equivalent.

MODEL OF BOND-IMPLIED CDS SPREADS

In order to analyse the relationship between bonds and CDS we need to establish a model which allows us to compare them directly. We can then use this model to establish a direct relationship between the asset swap spread, the bond yield spread and the CDS spread.

The default time τ of the reference credit is modelled as the first stopping time of a Poisson process with stochastic intensity $\lambda(t)$. This model is the standard approach for pricing credit vulnerable bonds and derivatives within a risk-neutral framework as it facilitates the fitting of the market term structure of CDS spreads or bond prices or both. We value a fixed coupon bond with N remaining full coupon payments at times $t_1, t_2, t_3, .., t_N = T$, each consisting of a coupon c paid with frequency f. We treat each coupon as a zero recovery payment made conditional on surviving to the coupon payment date. The appendix shows that subject to some simplifying assumptions concerning the independence of interest rates, the intensity process and the realised recovery rate, we can model the price of this bond as follows:

$$P(0) = c/f \sum_{n=1}^{N} Z(t_n)Q(t_n) + R\int_0^T Z(t)(-dQ(t)) + Z(T)Q(T) \quad (1)$$

where Q(t) is the risk-neutral survival probability of the reference credit to time t and Z(t) is the default-free discount factor. From this we can determine the following different spread measures.

Asset Swap Spread: We substitute the model bond price into the equation for the asset swap spread [O'Kane 2008] to give

$$S_{ASW}(0) = \frac{\widehat{P}(0) - P(0)}{\sum_{n=1}^{N} \Delta(t_{i-1}, t_i) Z(t_i)}$$

where the denominator is the present-value of 1 basis point paid on the floating leg of the asset swap, $\Delta(t_{i-1}, t_i)$ is the year fraction between successive payments in the basis convention for the floating leg of the asset swap, and

$$\widehat{P}(0) = c/f \sum_{n=1}^{N} Z(t_n) + Z(t_N)$$

is the full price of a Libor quality version of the same bond discounted on the Libor curve as represented by the discount factor $Z(t_i)$. These discount factors are those implied by a term structure of Libor deposit and swap rates.

The Yield Spread: Based on this simple model of the fixed coupon bond price we can determine the yield spread. This is calculated by solving for the yield-to-maturity for the T-year maturity credit risky bond using equation (1) and for the same-maturity default-free bond with the same maturity, and then subtracting to get the yield spread.

The CDS Spread: In the appendix we show that the model-implied CDS par spread for a euro denominated CDS can be written in terms of the risk-free discount factors and the survival curve as follows

$$S_{€}(0) = \frac{(1-R)\int_0^T Z(0,s)(-dQ(0,s))}{\frac{1}{2}\sum_{n=1}^{N} \Delta(t_{i-1}, t_i) Z(0, t_i)(Q(0, t_{i-1}) + Q(0, t_i))}. \quad (2)$$

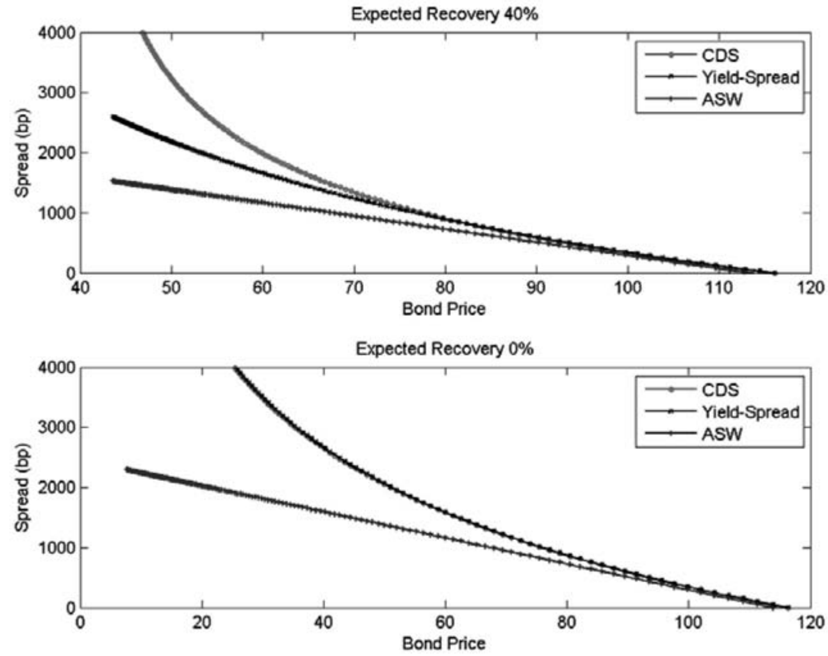
However the US dollar CDS spread needs to take into account the fact that conditional upon whether there is or there is not a credit event, there may be a change in the Eurodollar exchange rate between the two currencies. [Ehlers and Schonbucher 2006] have shown that the currency effect means that the ratio of the dollar and euro CDS spreads are given by

$$S_{\$}(0) = k \cdot S_{€}(0)$$

where k is the ratio of the immediate post-default eurodollar FX rate to the immediate pre-default eurodollar FX rate. The FX rate is quoted in units of number of euros per dollar. This means that if the market expects a devaluation of the euro following a credit event, the value of k will be greater than 1 and the dollar denominated CDS should have a higher par spread than the equivalent euro denominated CDS.

To examine the difference between these spread measures, we priced a 5-year bond with a 5% coupon in an environment where the default-free yield curve is assumed flat at 3% and the Libor risk-free curve is also assumed to be flat at 3.5%. We considered two cases - first an expected recovery rate of 40% and second an expected recovery of 0%. We chose the 40% recovery as this is the market standard recovery rate assumption used in the pricing of CDS $1 annualised coupon because it closely matches the expected recovery rate of senior unsecured debt, to which most CDS are linked, as found in rating agency default statistics. For example a recent report by [Qu et al. 2011] finds that the average US corporate recovery rate for senior unsecured debt from 1920-2010 was 36.7%. Another report by [Tudela et al. 2011] on sovereign debt recovery rates found that the average recovery rate from 1998-2008 was higher at about 53%. However

Figure 1: Comparison of the model-implied CDS, bond yield-spread and par asset swap spread measures as a function of the full price of a 5-year bond with a 6% coupon. We show this for an expected recovery of 40% (above) and 0% (below).



the number of defaults in this dataset was small and so the market continues to use 40% as its standard setting. This assumption does change when the credit becomes extremely distressed and market expectations about the post-default recovery are revealed in the prices of bonds.

We then assumed a flat term structure of default rates at a constant level h where $Q(0,t) = \exp(-ht)$. By varying the value of h we calculated the implied bond price, yield spread, asset swap spread and CDS spread. Initially we have assumed that $k = 1$, thereby ignoring any credit event contingent currency devaluation or appreciation.

The results are presented in Figure 1. When the expected recovery rate is 40% we find that as the bond price falls (and it cannot fall below 40), the CDS spread grows and asymptotically tends to infinity. At the same time the yield-spread and asset swap spread tend to different large but finite numbers. It is only in the limit of an expected recovery rate of zero that the yield-spread also tends to infinity and is almost indistinguishable from the CDS spread[3].

CHOICE OF SPREAD

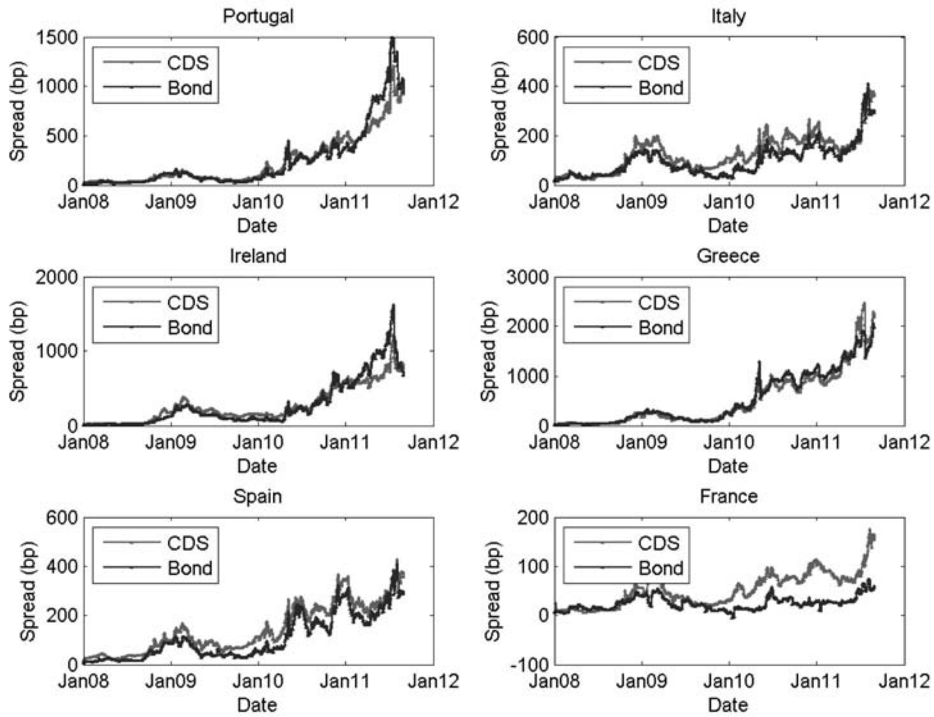
For our measure of bond credit spread, we choose the bond yield spread. The alternative would be to choose a Libor based credit spread such as the asset swap spread. However, as Libor is the reference rate for inter-bank lending, it embeds a compensation for the credit quality of the commercial banking sector. This means that sovereigns deemed to have a better credit quality than the banking sector will trade with a negative asset swap spread; however the comparable CDS spread is always floored at zero. Also, movements in the asset swap spread of a sovereign debt are not purely reflective of the sovereign's credit risk but of its credit risk relative to the banking sector. An increase in both would therefore be cancelled to some extent by a Libor-based spread measure.

We use the yield-spread of the bond measured as the yield of the bond minus that of the same maturity default-free bond, for which we proxy German bonds. The advantage of the yield-spread is that it is a measure of credit risk relative to the high credit quality German yield. The size of the German bond market also means that there is only a small liquidity premium. To get a sense of the difference in perceived credit quality between Libor and the German yield we note that the swap spread – the 5-year Libor swap rate minus the yield on a 5-year German government bond - averaged 51bp and varied between 14bp and 106bp over the period January 2008 to September 2011. The average German 5-year yield was 2.55% while the average 5-year swap rate was 3.06%.

DATA DESCRIPTION

Our analysis focuses on the periphery group of Eurozone countries which have been impacted by their high levels of debt and low levels of growth. Now known as the PIIGS, these are Portugal, Ireland, Italy, Greece

Figure 2: Evolution of the 5Y CDS and Bond spreads over the sample period for Portugal, Italy, Ireland, Greece, Spain and France.



and Spain. We also include France in our dataset as a reference. Germany is also included. However because we have chosen to use the German yield as the effective default-free rate in calculating the bond yield spread, we do not report any results for Germany since its yield spread is by definition zero.

In the following we use daily close prices for CDS and bonds for the period 1 January 2008 to 1 September 2011. We focused on the 5-year maturity in both the CDS and bond market as this is the maturity point where liquidity in the CDS market is maximal. We used Bloomberg as our CDS and bond data source. We used mid-market yield levels at the 5-year maturity over the period being analysed. These were the yields to maturities of the current on-the-run 5-year benchmark bonds in each market. Each time series contained 956 data points. The evolution of these spreads is also shown in Figure 2. Figure 3 shows the main properties of both the CDS and bond yield-spread data time series.

Figure 3: Descriptive statistics of the panel of CDS spread data and the bond yield-spread data used. Each data series contained 956 samples. All numbers are in basis points.

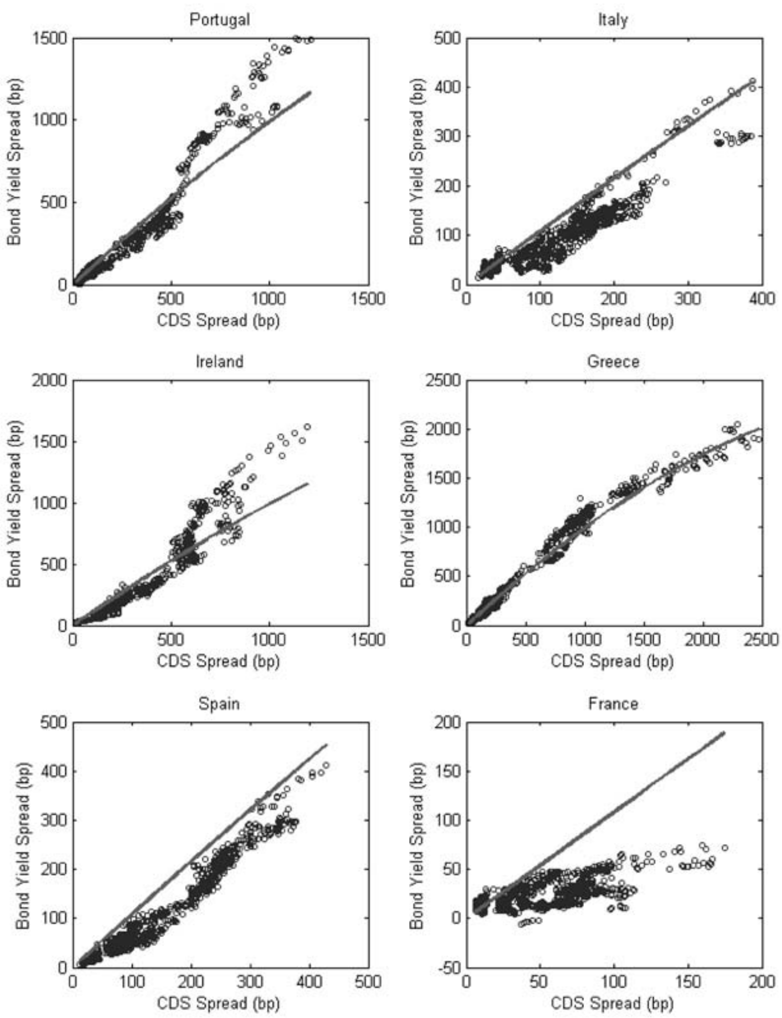
Country	CDS Spread Data				Bond Yield-Spread (vs Germany)			
	Min	Max	Average	Std Devn.	Min	Max	Average	Std Devn.
Portugal	14	1208	236	251	3	1497	245	314
Italy	17	388	128	73	15	412	99	66
Ireland	13	1192	275	240	10	1621	276	318
Greece	18	2477	510	541	21	2048	525	521
Spain	13	430	144	98	6	411	108	91
France	6	175	52	34	-6	74	25	13
Germany	5	91	34	19	0	0	0	0

EMPIRICAL ANALYSIS

We start by examining the contemporaneous relationship between the bond yield spreads and the CDS spreads in order to compare it to the model-implied relationship. A scatter plot of the bond yield spreads versus the CDS spreads is shown in Figure 4. We have also plotted the model-implied relationship. This assumes a recovery rate of 40% and a flat term structure of spreads and interest rates. We have also set $k = 1$ so that no currency effect is included. We also assume a risk-free rate of 2.5%, a 3.0% rate for 5-year Libor, an annual 4.0% coupon on the risk-free bond and an annual 6.0% coupon on the risky sovereign bond. The CDS coupon is paid quarterly using an Actual 360 convention and also includes the payment of coupon accrued at default which is the market standard.

Greece fits the theoretical curve well across the very broad range of spreads it has experienced and even presents some of the negative convexity implied by the model at high CDS spreads. In Portugal and Ireland the relationship is obeyed quite well up to about CDS spreads of 600bp. Beyond this spread level, it seems as though there is then a regime shift which causes the bonds to fall in price relative to the CDS by more than the model would imply. This could be because the bond market begins to become less liquid and this loss of liquidity causes bond prices to fall relative to CDS which continue to trade as before. It might also be because more information becomes available in the distressed prices of the bonds which cause market participants to revise downward the market-wide assumed expected recovery rate. This would increase the slope of the theoretical model. However, even with a recovery of 0%, we were unable to match the high positive slope of the relationship. Another explanation is that the market believes that a credit event may lead to the ejection of that economically weak periphery country from the eurozone resulting in an appreciation rather than a depreciation of the euro. This would be captured in our model by a value of $k < 1$. It is also worth noting that the

Figure 4: The bond yield-spread against the CDS spread for the PIIGS and France over the period January 2008 to September 2011. The line is the model-implied relationship assuming a recovery rate of 40%.



good model agreement for Greece could be implying that any currency impact of a Greek default is already priced into the exchange rate.

In Italy, Spain and France the CDS spreads appear too high for the bond yields compared to the model-implied relationship. This effect could be due to the 40% recovery rate assumption being too low. It could also be a "flight to safety" effect as euro-based holders of Greek, Irish or Portuguese debt switch into these larger core Eurozone countries as their bond holdings becomes more distressed. This demand would push down the bond yield for those countries relative to the CDS market. The high CDS spreads of Italy, Spain and France could also be due to a currency effect in which the market expects a devaluation of the euro if any of these sovereigns experience a credit event.

DYNAMICAL ANALYSIS

We analyse how the changes in bond yield spreads and CDS spreads are linked. We define the yield spread at time t to be given by $S_{Bond}(t) = y_{Country}(t) - y_{Germany}(t)$where $y_{Country}(t)$ is yield-to-maturity of the 5-year benchmark bond in that country and $y_{Germany}(t)$ is the yield-to-maturity for a 5-year benchmark German government bond. We define $S_{CDS}(t)$ to be the time t 5-year CDS spread linked to the same country. Our analysis is concerned with detecting causality in the changes of the CDS and bond yield-spreads. We define $\Delta S_{Bond}(t) = S_{Bond}(t) - S_{Bond}(t-1)$ and $\Delta S_{CDS}(t) = S_{CDS}(t) - S_{CDS}(t-1)$ to be the daily bond yield-spread change and CDS spread change. A starting point for analysing the dynamic relationship between bonds and CDS is to look at the cross-correlation between $\Delta S_{Bond}(t)$ and $\Delta S_{CDS}(t)$. We calculated the instantaneous pair wise correlation between the CDS and bond yield spread. This is shown in Figure 5 below – the row corresponding to a lag of zero.

We find that the relationship is quite strong with a correlation of 60% or more for all countries except France. We suspect that the low 28.3% correlation for France is due to a previously mentioned "flight to safety" effect which means that when France's CDS spreads are widening due to the contagion from the problems of the periphery countries, bond investors are often buying French debt, viewing France and Germany as the Eurozone's safe haven bond markets.

LAGGED CORRELATION

The first step in examining evidence of causality is to examine lagged correlation. We define the lagged cross-correlation as

$$\rho_{CDS,Bonds}(l) = corr(\Delta S_{CDS}(t), \Delta S_{Bond}(t+l)).$$

If this is positive and statistically significant, it implies that an increase in the CDS spread today will tend to result in an increase in the bond yield spread l days later. It is an indication that CDS market leads the bond market. When the lag l is positive, a positive correlation suggests that the CDS market leads the bond market. When the lag l is negative, a positive correlation suggests that the bond market leads the CDS market.

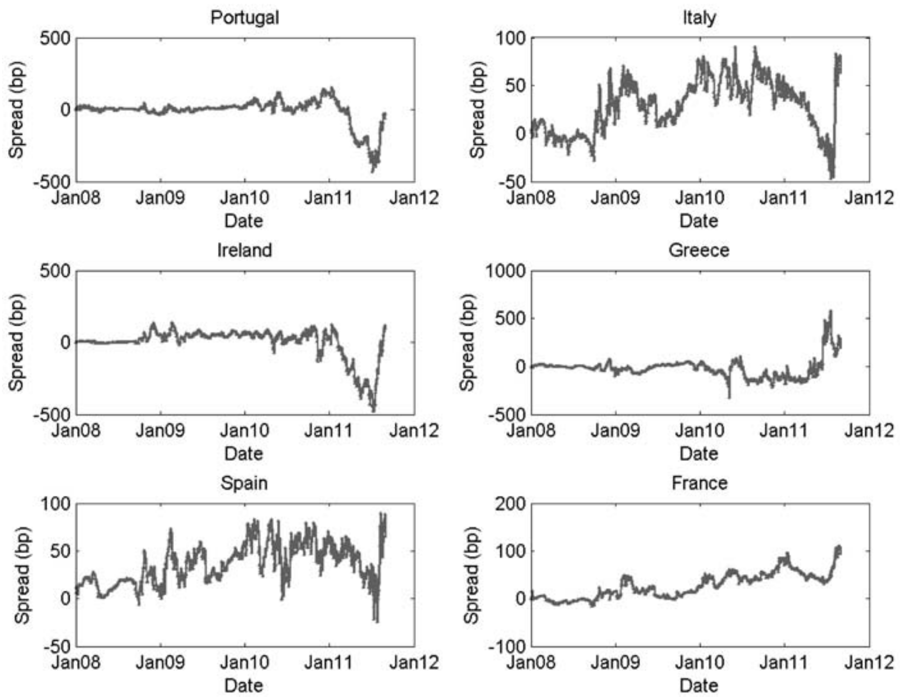
We see that cross correlations drop as we move to the one day lead/lag yet they are still statistically significant from zero. Beyond the one-day lead/lag, most of the cross-correlations are no longer statistically significant except for a few large and negative correlations in the case of Portugal, Spain and Italy at lags of up to 4 days.

The time series of both yield-spread and CDS spread changes exhibited a number of large daily movements or jumps, in both the positive and negative direction. We tested the extent to which our results were due to such jumps. We first recalculated these lagged correlations after capping all daily spread moves with a magnitude greater than some threshold, thereby removing reducing the impact of large jumps. We found that this reduced synchronous correlations. This was explained by an examination of the data which showed that there were a number of days[4] associated with Eurozone debt crisis on which CDS and bond markets moved synchronously and by a large amount. In the case of Greece, the synchronous correlation between CDS and bond spread changed fell from 71.5% to 51.7% when capping the maximum daily

Figure 5: Lagged cross-correlations between CDS spread changes and bond yield-spread changes by country and lag.

Lag (days)	Portugal	Italy	Ireland	Greece	Spain	France
– 4	– 20.9	– 4.0	– 2.3	– 7.2	– 7.1	5.1
– 3	0.9	– 4.9	5.5	– 0.9	– 11.5	0.1
– 2	3.7	– 10.6	5.6	4.3	– 11.4	3.2
– 1	28.0	19.3	31.1	18.4	15.0	10.1
0	72.5	70.4	62.4	71.3	72.3	28.3
1	36.7	20.0	23.2	17.8	28.9	8.8
2	7.1	– 10.6	11.5	2.6	– 6.4	-1.4
3	– 11.8	– 3.6	– 4.3	– 5.7	– 13.7	3.9
4	– 18.2	– 10.7	– 2.8	– 8.4	– 16.6	2.6

Figure 6: The CDS basis over time quoted as the CDS spread minus the bond yield spread.



jump at 50bp. It also increased the 1-day lagged cross correlation from around 18% to 28%. As a second test, we calculated the lagged correlations but only allowing spread movements with a magnitude greater than some threshold, thereby only including jumps. We found that selecting for jumps only increased the zero lag cross-correlations for the reasons described previously. In both sets of tests, we found that the lagged correlation numbers did change, but that the sign and magnitude of the auto and cross-correlation statistics did not change materially when capping jumps or only including jumps. This suggested that similar underlying market dynamics were driving both the small and large spread movements.

GRANGER CAUSALITY TEST

If the measured lagged correlations are positive and statistically significant from zero, we can infer that some temporal relationship exists. To examine this in greater detail we can test for the presence of Granger causality. A Granger causality test is a more powerful indicator of causality than the lagged cross-correlation as it determines whether there is a flow of information from one variable to another, and the direction in which it travels.

Before examining Granger causality we must determine whether or not the CDS and bond yield spread processes exhibit co-integration. This occurs when two integrated I(1) processes can be combined linearly to create a process with a long term equilibrium relationship. Figure 6 shows the evolution of the CDS basis defined as $S_{CDS}(t) - S_{Bond}(t)$ through time for each of the Eurozone sovereigns. We wish to test whether this time series, or any other linear combination of the CDS and bond yield spreads, exhibits a long term relationship. Existence of co-integration would suggest the use of an error correction model (ECM) as a more appropriate model choice for the dynamics of the CDS and bond spreads.

To test for co-integration we must first determine whether or not the CDS and bond yield spread processes used in this analysis each have a unit root, i.e. that they are non-stationary integrated processes. We did this using the Augmented Dickey-Fuller (ADF) statistic and the results are shown in Figure 7. The more negative the ADF statistic, the stronger the rejection of the hypothesis that the process has a unit root. We conclude from these results that the unit root hypothesis was not rejected for any of the CDS or bond yield-spread processes.

We then used the augmented Dickey-Fuller (ADF) statistic to test for co-integration on a linear combination of CDS and bond-yield spreads. Co-integration requires that there is a linear combination of the two spread processes which does not have a unit root and so is therefore stationary, i.e. unlike the previous test, we would need to reject the ADF null hypothesis of a unit root. For this to occur the ADF t-statistic must be greater than the critical value at some confidence level. From our analysis, we found that the ADF t-statistics for all countries were negative as shown in Figure 8. However, given that the 1% critical value was -3.88 and the 10% critical value was -3.04, the null hypothesis of a

Figure 7: ADF test results for a unit root for the CDS spreads and bond yield spreads. The null hypothesis of a unit root in each of these processes is not rejected in all cases. Note that the 10% critical threshold is -2.584.

	Augmented Dickey-Fuller t-statistic	
Country	CDS Spreads	Bond Yield Spreads
Portugal	1.479	0.910
Italy	0.810	0.392
Ireland	0.298	– 0.631
Greece	2.231	2.039
Spain	0.803	0.111
France	1.154	– 0.743

unit root was rejected at the 90% confidence level by France and at the 99% confidence level by Spain. These results suggest that the relationship between Spanish CDS and bond yield-spreads and between French CDS and bond yield-spreads may exhibit cointegration. At these confidence levels, the possibility of cointegration is ruled out for the other countries. We therefore did not build an ECM but performed a standard Granger causality test on all of the countries. We included France and Spain in this, although we must caveat this by acknowledging that the possible existence of cointegration in their time series may produce spurious results when a Granger causality test is applied.

The Granger causality test makes it possible to test for causality in both directions – i.e. from bonds to CDS and vice-versa. The first step of the Granger causality test was to determine the number *m* of lags for the CDS spread changes time series by regressing $\Delta S_{CDS}(t)$ against its lagged values back to $\Delta S_{CDS}(t-m)$ and choosing the regression with the number of lags which optimises the Bayesian Information Criterion (BIC). We then determined the optimal number of lags n of time series $\Delta S_{Bond}(t)$ by regressing $\Delta S_{CDS}(t)$ against its m lagged values and the n lagged values of $\Delta S_{Bond}(t)$. Once again we chose the regression with the value of n which optimises the BIC. In all optimisations we allowed a maximum lag of 5 days. The underlying model equation is:

$$\Delta S_{CDS}(t) = \sum_{i=1}^{m} \alpha_i \Delta S_{CDS}(t-i) + \sum_{j=1}^{n} \beta_j \Delta S_{Bond}(t-j) + \varepsilon_1.$$

We then tested the null hypothesis which is that the lagged values of $\Delta S_{CDS}(t)$ do not Granger cause $\Delta S_{CDS}(t)$. Formally, this is the test that $\beta_1 = \beta_2 = ... = \beta_n = 0$. A

Figure 8: The co-integrated Dickey-Fuller t-statistic for CDS and bond yield-spreads.

Portugal	Italy	Ireland	Greece	Spain	France
– 2.59	– 2.95	– 2.67	– 2.70	– 3.92	– 3.04

Figure 9: Results of the Granger test showing the F-statistic, p-statistic and the optimal number of lags based on the BIC criterion.

Null Hypothesis	CDS do not lead bonds				Bonds do not lead CDS			
Country	F-statistic	P-statistic (%)	Optimal Lags m/n	Accept/ Reject	F-statistic	P-statistic (%)	Optimal Lags m/n	Accept/ Reject
Portugal	33.7	0.0	4/3	R	16.5	0.0	5/4	R
Italy	5.3	2.2	4/1	A	10.8	0.0	2/2	R
Ireland	16.3	0.0	5/3	R	19.2	0.0	1/1	R
Greece	32.6	0.0	1/1	R	2.92	8.8	1/1	A
Spain	27.1	0.0	2/1	R	5.49	1.9	5/1	A
France	4.76	3.1	1/1	A	47.9	0.0	5/1	R

deviation from this with statistical significance as measured using an F-statistic allows us to accept or reject this null hypothesis. We set a significance level of 0.1% for rejection of the hypotheses. We then repeated this exercise with the model equation

$$\Delta S_{Bond}(t) = \sum_{i=1}^{m} \alpha_i \Delta S_{Bond}(t-i) + \sum_{j=1}^{n} \beta_j \Delta S_{CDS}(t-j) + \varepsilon_2$$

in an attempt to see if CDS spread changes Granger cause bond spread changes. The results are shown in Figure 9.

A rejection of a hypothesis to 99.9% significance is signalled by a p-statistic lower than 0.1%. Our results show that there are three categories of country and mutual causality relationship. We find that: (i) in Greece and Spain, CDS spread changes Granger cause changes in the bond yield spreads but bond yield changes do not Granger cause CDS spread changes; (ii) in Italy and France, bond yield spread changes Granger cause CDS spread changes but CDS spread changes do not Granger cause bond yield spread changes; and (iii) in Portugal and Ireland, CDS spread changes and bond yield spread changes lead each other, a result which can be interpreted as a feedback. We note that many of the Granger causing lags represented by the value of n were at just one day suggesting that any information transfer effect is short-lived.

Conclusions

A theoretical model-based analysis of the relationship between CDS spreads and bond yield spreads shows that CDS spreads generally trade wide to bond yield spreads, especially in times of distress, due to differences in the format of CDS and bonds, and due to the different definitions of these spreads. Therefore a positive CDS basis, measured as the par CDS spread minus the bond yield spread, is not in itself a sign that the CDS and bond markets are dislocated and certainly not a confirmation of speculative activity in the form of CDS protection buyers.

Empirical analyses of the relationship between CDS spreads and bond yield-spreads show that the market only obeys the theoretical model implied relationship in a very approximate way. One reason why this relationship is not strictly observed is that the model-implied arbitrage is not even approximately a tradable arbitrage when the credit risky bond is pricing away from par. This means that market participants will only act on theoretical arbitrages when dislocations become large enough to make the risk-return profile of the trade attractive. As a result, divergences between the two markets may persist through time, even if they have no economic basis or the initial cause of the basis goes away.

An additional factor which could be material in causing CDS and bond spreads to diverge, and which we have discussed, is the denomination of market-standard CDS contracts on European sovereign debt in US dollars. In this case the ratio of the US dollar CDS spreads and the bond-implied euro CDS spreads may indicate the market's expectations for the impact of a sovereign default on the Eurodollar exchange rate. Ignoring other potential factors, these tentatively suggest that the market is pricing in an appreciation of the euro if Greece, Portugal or Ireland default, and a depreciation of the euro if Italy, Spain or France default. However we caveat the results for France since we believe that the CDS basis contains a significant "flight to safety" effect.

We find that lagged values of changes in the CDS spread and changes in the bond yield spread exhibit both autocorrelation and cross-correlation, with the greatest effect occurring at a lag of one day. These correlations were especially significant for the sovereigns - Portugal, Spain and France - which experienced the greatest spread widening over the sample period.

We reject the hypothesis that CDS spreads and bond yield spreads are co-integrated for Greece, Italy, Portugal and Ireland. This runs counter to previous work on the corporate CDS basis by [Blanco et al. 2005] who found that most CDS spreads and bond yield spreads are co-integrated, and that where they are not, the explanation is probably due to a valuable cheapest to deliver (CTD) option which is embedded in the CDS. Although this may also be an effect here, we do not believe that the CTD option is the explanation for all of the Eurozone sovereigns since it would only explain a positive basis (the CTD will make the CDS spread wider as the protection buyer is long the CTD option) whereas we found a mixture of a negative basis in Portugal and Ireland and a positive basis in Italy, Spain and France. Instead we suggest that the lack of co-integration is due to several factors including the market expected change in the Eurodollar exchange rate following a sovereign default.

Granger causality tests can give us indications of the direction of any information flow and the results of our Granger causality tests were mixed. They suggested that the dominant direction of causality is from CDS to bonds for Greece, but from bonds to CDS for France and Italy, while Ireland and Portugal exhibited Granger causality in both directions, implying a feedback system. We emphasise that a positive test for Granger causality is not evidence of true causality. However while a negative test would rule out the hypothesis of true causality, a positive test tells us that we cannot currently reject such a hypothesis.

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1 See "EU Ban on 'naked' CDS to become permanent', *Financial Times*, October 19, 2011.

2 If P is the price of the bond and R is the expected recovery rate, then protection of principal means that the face value of CDS protection needed equals $G = (P - R)/(1 - R)$.

3 Note that in order to obtain this very close agreement between CDS spread and yield-spread, we had to set the payment frequency of the bond equal to that of the CDS for which the standard is quarterly.

4 These included the 27th April 2010 – the period following Moody's downgrade of Greece, the 10th May 2010 when the EU and IMF agreed to an emergency fund, and the 20th to 21st of July 2011, the time of the initial Greek restructuring deal.

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Appendix

Valuation of a Fixed Coupon Bond

The default time τ of the reference credit is modelled as the first stopping time of a Poisson process with stochastic intensity $\lambda(t)$. This model is the standard approach for pricing credit vulnerable bonds and derivatives within a risk-neutral framework as it facilitates the fitting of a term structure and the imposition of an exogeneous expected recovery rate at the time of default.

We value a fixed coupon bond with N remaining full coupon payments at times $t_1, t_2, t_3, .., t_N = T$, each consisting of a coupon c paid with frequency f. We treat each coupon as a zero recovery payment made conditional on surviving to the coupon payment date. Recovery $R(\tau)$ is paid as a fixed percentage of par at the time of default. The bond price is given by

$$P = c/f \sum_{n=1}^{N} Z(0,t_n) E_0^Q \left[\exp\left(-\int_0^{t_n} (r(s)+\lambda(s))\,ds \right) \right] + \int_0^T Z(0,t) E_0^Q \left[\lambda(t) R(t) \exp\left(-\int_0^t (r(s)+\lambda(s))\,ds \right) \right] dt$$

$$+ E_0^Q \left[\exp\left(-\int_0^T (r(s)+\lambda(s))\,ds \right) \right]$$

where $r(t)$ is the default-free short-rate process at time t. We take the expectation in the risk-neutral measure. Assuming independence between interest rates, the intensity process and the realised recovery rate, we can write the bond price as

$$P = \frac{c}{f} \sum_{n=1}^{N} Z(0,t_n) Q(0,t_n) + R \int_0^T Z(0,t)(-dQ(0,t)) + Z(0,T) Q(0,T)$$

where $R = E_0^Q \left[R(\tau) \right]$ and $Z(0,t) = E_0^Q \left[\exp\left(-\int_0^t r(s)\,ds \right) \right]$ is the risk-free discount factor and $Q(0,t) = E_0^Q \left[\exp\left(-\int_0^t \lambda(s)\,ds \right) \right]$ is the survival probability of the reference credit to time t in the risk-neutral measure.

Determination of the CDS Par Spread

The premium leg of a CDS is the regular payment of an annualised coupon $S(0)$ which occurs at times $t_1, t_2, t_3, .., t_N = T$ ending at either contract maturity time T, or the time of a credit event. The premium leg value is therefore given by (see [O'Kane 2008] for details)

$$S(0) \sum_{n=1}^{N} \Delta(t_{i-1}, t_i) Z(0,t_i) Q(0,t_i)$$

Following a default the market convention is to include an additional payment of the coupon which has accrued. Adjusting for this, the value of the premium leg becomes $\frac{S(0)}{2} \sum_{n=1}^{N} \Delta(t_{i-1}, t_i) Z(0,t_i) \left(Q(0,t_{i-1}) + Q(0,t_i) \right)$

The protection leg of the CDS is the payment of par minus the recovery rate at the time of a credit event. The present value of this is given by

$$(1-R) \int_0^T Z(0,s)(-dQ(0,s))$$

The par spread is the coupon on the CDS contract which sets its value to zero. We then obtain equation (2).